

Hairpin-Branes and Tachyon-Paperclips in Holographic Backgrounds

Vasilis Niarchos

Centre de Physique Théorique, École Polytechnique

CNRS, 91128 Palaiseau, France

niarchos@cpht.polytechnique.fr

Abstract

D-branes with a U-shaped geometry, like the D8 flavor branes in the Sakai-Sugimoto model of QCD, are encountered frequently in holographic backgrounds. We argue that the commonly used DBI action is inadequate as an effective field theory description of these branes, because it misses a crucial component of the low-energy dynamics: a *light* complex scalar mode. Following an idea of Erkal, Kutasov and Lunin we elaborate on an effective description based on the *abelian* tachyon-DBI action which incorporates naturally the non-local physics of the complex scalar mode. We demonstrate its power in a context where an explicit worldsheet description of the open string dynamics exists —hairpin-branes in the background of NS5-branes. Our results are relevant for the holographic description of chiral symmetry breaking and bare quark mass in QCD and open string tachyon condensation in curved backgrounds.

May, 2010

Contents

1. Introduction	1
2. An abelian effective field theory description of the $D\overline{D}$ system in flat space	6
3. Hairpin-branes in holographic backgrounds	11
3.1. Lightning review of the Sakai-Sugimoto model	11
3.2. A Little String Theory analog of the Sakai-Sugimoto model	13
3.3. Open string theory in the near-horizon region of NS5-branes	15
4. Effective field theory description of D-branes intersecting NS5-branes	19
4.1. The TDBI action and its equations of motion	19
4.2. Constraining the tachyon potential: $ T \ll 1$	21
4.3. Constraining the tachyon potential: $ T \gg 1$ and paperclip asymptotics	22
4.4. Normalizable vs non-normalizable modes of the $D\overline{D}$ tachyon	26
4.5. The solution near the turning point	27
4.6. Numerical results	29
4.7. Concluding comments on the role of the DBI action	34
5. Flavor D8-branes in the Sakai-Sugimoto model	35
6. Conclusions	37
A. Worldsheet boundary interactions for hairpin-branes on the cigar	38
B. Tachyon-paperclips and the $D\overline{D}$ tachyon in general holographic backgrounds	40
C. Further numerical results on the relation $\mu_T(\rho_0)$: the case of generic k	42

1. Introduction

An important problem in holographic discussions of large- N gauge theories is how to describe efficiently the properties of systems that exhibit flavor chiral symmetry breaking (χ sB). By now a variety of holographic setups that exhibit chiral symmetry breaking is known. In the quenched approximation the flavor degrees of freedom are realized by a set of D-branes that extend in the radial direction of the holographic background. In some of these setups, the breaking of the flavor chiral symmetry occurs as a D-brane reconnection process.¹ To be concrete, let us consider a few well known examples.

The holographic physics of chiral symmetry breaking was discussed already in the early work [2]. One of the examples in that paper, based on a D3-D7-D7' system in type IIB string theory, realizes an $\mathcal{N} = 2$ gauge theory that can be viewed as $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory with $2N_f$ extra hypermultiplets. The N_f D7 and N_f D7' branes, which are

¹The situation is different, for instance, in the D4-D6 system of Ref. [1].

oriented in orthogonal directions —the D7's are placed at $z = X^8 + iX^9 = 0$ and the D7's at $w = X^6 + iX^7 = 0$ —, carry a non-chiral $U(N_f) \times U(N_f)$ flavor symmetry group. In gauge theory this group is broken explicitly to its diagonal with a mass deformation for the fundamental hypermultiplets. In the dual $\text{AdS}_5 \times S^5$ background the D7-D7' pairs reconnect to wrap a smooth curve of the form $zw = \varepsilon \neq 0$.

The Sakai-Sugimoto model of large- N QCD is another characteristic example [3, 4]. This model is based on a D4-D8- $\overline{\text{D8}}$ system where a $U(N_f) \times U(N_f)$ flavor symmetry group is realized on a stack of N_f D8- $\overline{\text{D8}}$ pairs. The antiparallel D8 and anti-D8 branes in each pair are separated in a transverse direction. At large 't Hooft coupling, where the supergravity description of the D4-branes is appropriate, the D8- $\overline{\text{D8}}$ pairs reconnect to N_f D8-branes with a U-shaped geometry. This feature reproduces naturally the spontaneous chiral symmetry breaking that occurs in the dual QCD-like gauge theory.

Qualitatively similar open string physics takes place in a non-supersymmetric Little String Theory (LST) version of the Sakai-Sugimoto model (for a review of LST see [5]). Replacing the D4-branes by k NS5-branes we obtain a stack of ℓ $\text{D}p\text{-}\overline{\text{D}p}$ pairs ($1 \leq p \leq 5$) intersecting orthogonally the NS5-branes. In the gravitational dual of this system the $\text{D}p\text{-}\overline{\text{D}p}$ pairs reconnect to ℓ $\text{D}p$ -branes with a U-shaped embedding in the curved near-horizon background of the NS5 branes. Because of their U shape, these branes are frequently called hairpin-branes.

Each of these setups requires a detailed understanding of the open string physics on the reconnected hairpin flavor branes. The resolution of fundamental questions about the flavor sector of the strongly coupled dual gauge theory depends on this understanding, *e.g.* questions about the order parameter of χsB , bare quark masses, the structure of the mesonic spectra, *etc.* Lacking an explicit solution of string theory, in many cases we rely heavily on effective field theory descriptions. The standard description of low-energy dynamics in open string theory is the Dirac-Born-Infeld (DBI) action. In this paper we will argue that this description is inadequate for the kind of D-brane systems that we discussed above.

In the first example of the D3-D7-D7' system it was already pointed out in [2] that the mass deformation that breaks the flavor chiral symmetry explicitly corresponds to giving a vacuum expectation value (vev) to a bifundamental hypermultiplet. This hypermultiplet arises as a massless mode in the NS— sector of an open string that is localized in the six dimensional intersection of the orthogonal D7 and D7'-branes. An effective action that includes the non-linear completion of the six dimensional hypermultiplet is needed to describe the full dynamics of this system [2].

A similar effect occurs in the second example, the Sakai-Sugimoto model. It is clear holographically that the chiral symmetry breaking that takes place in this case should be attributed again to the vev of a complex scalar field that belongs in the bifundamental representation of the $U(N_f) \times U(N_f)$ flavor symmetry group. This field arises as a ground state in the NS– sector of open strings that stretch between the D8 and anti-D8 branes. In flat space this mode is tachyonic when the separation of the D8 and anti-D8 branes L is comparable to the string scale ℓ_s and highly massive when $L \gg \ell_s$. Because of this, one may be tempted to conclude that there is a similar regime in the Sakai-Sugimoto model where the ground state in the NS– sector of the D8– $\overline{\text{D8}}$ strings is also highly massive and therefore irrelevant for the infrared physics. We will argue that this intuition from flat space is in fact misleading and that this mode can be light in curved backgrounds even when $L \gg \ell_s$. In holographic setups the mass formula of this mode can get a large negative contribution from the non-trivial profile of the wavefunction in the radial direction that cancels the large positive contribution related to the separation L . A similar picture was advocated in [6]. Here we set up an effective description of open string dynamics that incorporates naturally this effect and corroborates this picture.

In the third example, the NS5- $Dp\text{--}\overline{Dp}$ system, we can exhibit this effect very explicitly by solving the open string theory on the $Dp\text{--}\overline{Dp}$ branes with the use of α' -exact methods of worldsheet conformal field theory (CFT). We can identify the NS– sector mode of interest, compute its mass and see how it condenses on the Dp -branes.²

Our purpose in this paper is to set up an effective field theory description of the low-energy open string dynamics on U-shaped reconnected $Dp\text{--}\overline{Dp}$ systems of the type presented in the second and third examples above. Besides the transverse scalars and gauge fields this action must also capture the dynamics of a complex scalar field in the bifundamental of the flavor gauge group that comes from a long open string of the $Dp\text{--}\overline{Dp}$ system. Following the standard nomenclature of $D\overline{D}$ systems we will call this mode a tachyon, but it should be remembered that it gives rise to a massless field in the systems that we will examine. Finding a well motivated, efficient description of this system is currently a largely open problem.

Previous attempts [8, 9] to incorporate this bifundamental mode into the analysis of the

²The automatic appearance and relevance of a non-vanishing vev for a normalizable mode of the bifundamental scalar was also emphasized in Ref. [7] in a related context—the context of the non-critical near-horizon background of two orthogonal NS5-branes, where probe D3 and D5-branes were used to engineer the four-dimensional $\mathcal{N} = 1$ supersymmetric QCD. In that case the spacefilling flavor D5-branes are T-dual to D4 hairpin-branes and the appearance of a non-vanishing vev for the bifundamental scalar has a natural interpretation in SQCD.

Sakai-Sugimoto model were based on a non-abelian tachyon-DBI (TDBI) action that has been proposed [10, 11] to describe the physics of $D\overline{D}$ systems in flat space. Unfortunately, it is unclear, already in flat space, to what extent these non-abelian actions provide a good effective field theory description of open string physics. Moreover, it is unclear in the application of Refs. [8, 9] how one incorporates the non-local physics of the bifundamental field.³

In what follows, we will take a different route based on a recent proposal for a new effective description of the dynamics of $D\overline{D}$ systems in flat space put forward by Erkal, Kutasov and Lunin in Ref. [14]. The basis of this approach is the *abelian* tachyon-DBI action (we will concentrate on a single $Dp\text{-}\overline{Dp}$ pair and set $\alpha' = 1$)

$$\mathcal{S} = - \int d^{p+2}x V(T) \sqrt{-\det A} , \quad (1.1a)$$

$$A_{ab} = \eta_{ab} + \partial_a X^I \partial_b X^I + 2\pi F_{ab} + \partial_a T \partial_b T , \quad (1.1b)$$

$$V(T) = \frac{\tau_{p+1}}{\cosh(\alpha T)} \quad (1.1c)$$

that describes the dynamics of a *real* tachyon T on a non-BPS $D(p+1)$ -brane with tension τ_{p+1} . The constant α that appears in the tachyon potential (1.1c) is 1 for the bosonic string and $\frac{1}{\sqrt{2}}$ for the type II string. The $Dp\text{-}\overline{Dp}$ system arises in this description as a kink-antikink solution. The complex $D\overline{D}$ tachyon, the $U(1) \times U(1)$ gauge field and the extra transverse scalars are emergent degrees of freedom. We will review the basic features of this formalism in section 2. The formalism involves a particular type of solutions of the action (1.1) with a ‘paperclip’ profile in an extended spacetime that includes T as a fictitious extra spacetime coordinate (the second ingredient of the title of the present work —‘tachyon-paperclips’— refers to this type of solutions).

In contrast to the proposed non-abelian DBI actions for the $D\overline{D}$ tachyon, the action (1.1) has been derived from open string theory in a well-defined decoupling limit and is known to describe small fluctuations around the rolling tachyon background (more precisely, around the ‘half S-brane’) [15, 16].⁴ In this derivation the rolling tachyon solution plays the same

³This problem is absent in the phenomenological five-dimensional setup of Ref. [12] where the branes are coincident with the anti-branes. For a more recent phenomenological application of the Sen action [10] in coincident brane-antibrane systems see [13].

⁴Strictly speaking, the derivation of [15, 16] was performed for vanishing velocities and gauge field strengths. The more general action (1.1) passes a number of non-trivial tests [10, 14] and is believed to be the correct way to incorporate the transverse scalars and abelian gauge field.

role that solutions with constant electromagnetic field and/or velocity play in the case of the usual DBI action.

In the rest of this paper, we would like to treat the action (1.1) as a well motivated toy field theory action for open string dynamics and apply it to general closed string backgrounds. Part of our exercise will be to verify that it produces results consistent with knowledge from open string theory in curved (holographic) backgrounds (NS5-brane backgrounds in this work). We will assume the validity of the general form of the action (1.1a), with the obvious incorporation of the induced metric in A_{ab} , but will keep the precise form of the tachyon potential free allowing in this factor the possibility of a background dependence. We will discuss the motivation for leaving such a possibility open. In the context of the NS5-brane system we will present a tachyon-paperclip solution that reproduces anticipated features of string theory but requires a modification of the potential (1.1c).

The main lessons of this paper can be summarized as follows:

- (1) We will see explicitly in section 3, in the context of the NS5- $Dp\overline{Dp}$ system, how a complex scalar mode from a long open string arises in the low-lying perturbative spectrum of the open string theory on the U-shaped Dp -branes. On the worldsheet, a marginal boundary interaction of this mode is dual to a boundary interaction that captures the geometric bending of the brane [17, 18, 19, 20]. Both interactions are turned on simultaneously and the geometric reconnection of the brane is intimately related with the condensation of a normalizable mode of the bifundamental complex scalar field. There are no branes, in particular, with vanishing condensate of the complex scalar mode whose dynamics is captured solely by the DBI action.
- (2) In section 4 we set up an effective tachyon-DBI action that captures efficiently the interacting dynamics of the complex scalar field, the transverse scalars and abelian gauge field and reproduces key features of the exact string theory results in the NS5- $Dp\overline{Dp}$ system, *e.g.* the exact CFT dependence of the boundary cosmological constant on the hairpin-brane turning point. In this description the duality of the worldsheet boundary interactions of the previous item acquires an intuitive geometric interpretation. The non-local nature of the complex scalar field is incorporated naturally. In our treatment of the tachyon-DBI action the tachyon potential is a free function. We find an asymptotic tachyon-paperclip solution that reproduces some of the exact string theory information and solves the equations of motion with a modified potential, *i.e.* a potential different from the $1/\cosh$ one that has been derived in flat space.

- (3) We clarify the role of the DBI action in this description. The deviations from the DBI solution are supported near the turning point of the hairpin-brane. We determine analytically the behavior of the tachyon-DBI solution near the turning point.
- (4) We study how one incorporates the non-normalizable mode of the bifundamental complex scalar field in the asymptotics of the solution. In the Sakai-Sugimoto model this mode is responsible for giving bare mass to the quarks. An issue in previous discussions of the Sakai-Sugimoto model was how to incorporate this mode without violating the boundary condition that the $D\bar{D}$ tachyon vanishes at the asymptotic infinity. We will see that our effective description gets around this problem in an interesting way.

Our ultimate goal is to import these lessons in more general contexts where no a priori worldsheet control is available. We are particularly interested in the Sakai-Sugimoto model and its implications for the strong coupling dynamics of QCD. A preliminary discussion of this system appears in section 5. A more thorough examination of the abelian TDBI description of the Sakai-Sugimoto model is currently under investigation [21].

2. An abelian effective field theory description of the $D\bar{D}$ system in flat space

In this section we review the key points of a new effective description of the $D\bar{D}$ system in flat space that was proposed in [14]. This effective description will be the basis of our subsequent analysis of hairpin-branes in holographic backgrounds.

Consider the action (henceforth we set $\alpha' = 1$)

$$\mathcal{S} = - \int d^{p+1} \sigma e^{-\Phi} V(T) \sqrt{-\det(G_{ab} + 2\pi F_{ab})} \quad (2.1)$$

where σ^a ($a = 0, 1, \dots, p$) are worldvolume coordinates,

$$G_{ab} = g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \partial_a T \partial_b T \quad (2.2)$$

is the induced metric, F_{ab} is the field strength of an abelian gauge field and $g_{\mu\nu}(X)$, $\Phi(X)$ are the spacetime metric and dilaton respectively. Formally, this is the DBI action for a p -brane propagating in a ‘fictitious’ $(10+1)$ -dimensional spacetime with an extra spatial coordinate T and metric

$$ds^2 = g_{\mu\nu} dX^\mu dX^\nu + dT^2. \quad (2.3)$$

From this point of view the potential $V(T)$ appears as a T -dependent contribution to the dilaton field.

Ref. [14] observes that the action (2.1), with potential $V(T)$ given by (1.1c), provides a unified description of BPS and non-BPS branes in flat space. The non-BPS Dp -brane is obtained by orienting the worldvolume perpendicular to T . Indeed, by choosing the static gauge $X^a = \sigma^a$ ($a = 0, 1, \dots, p$) one recovers the TDBI action (1.1). The fields of this description are the worldvolume gauge field, the physical space transverse scalars, and the tachyon field T .

The BPS $D(p-1)$ -brane is obtained by choosing a different orientation of the $(p+1)$ -dimensional worldvolume in the extended spacetime—an orientation *parallel* to T . Adopting the static gauge $X^a = \sigma^a$ ($a = 0, 1, \dots, p-1$), $T = \sigma^p$ we recover an action for the transverse scalars and $U(1)$ gauge field. With a T -dependent transformation one can set the T -component of the gauge field, A_T , to zero. This leaves a residual symmetry of T -independent gauge transformations. The remaining fields are all functions of both σ^a and T , but Ref. [14] shows that the general T -dependent configuration is non-normalizable and does not describe open string excitations. In fact, one can argue that such configurations describe closed string excitations in accordance with the open string completeness proposal of Sen [22]. The bottom line of this discussion is that one should consider T -independent profiles of the fields X^μ, A_a . Then, the action (2.1) becomes

$$\mathcal{S} = - \left(\int dT V(T) \right) \int d^p \sigma e^{-\Phi} \sqrt{-\det(g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + 2\pi F_{ab})} . \quad (2.4)$$

In flat space, with the tachyon potential (1.1c), the BPS brane tension is

$$\tau_{p-1}^{\text{BPS}} = \int dT V(T) = \sqrt{2\pi} \tau_p^{\text{non-BPS}} \quad (2.5)$$

and (2.4) reduces nicely to the DBI action for a BPS $D(p-1)$ -brane. It is possible to generalize this discussion to include worldvolume fermions and Wess-Zumino couplings [14]. Since neither of these extra features will be important for our purposes we will not consider them here explicitly.

We will assume that the action (2.1) is a sensible starting point for the effective field theory description of BPS and non-BPS D-branes in general spacetime backgrounds with a background-dependent tachyon potential $V(T)$. In section 4 we will verify this assumption in a class of curved backgrounds where an exact string theory description is available.

So far we have seen how standard BPS or non-BPS D-branes in flat space are reproduced by trivial planar solutions of the action (2.1). It is interesting to explore the physical meaning of more general solutions with different orientations/shapes in the extended $(10+1)$ -dimensional spacetime.

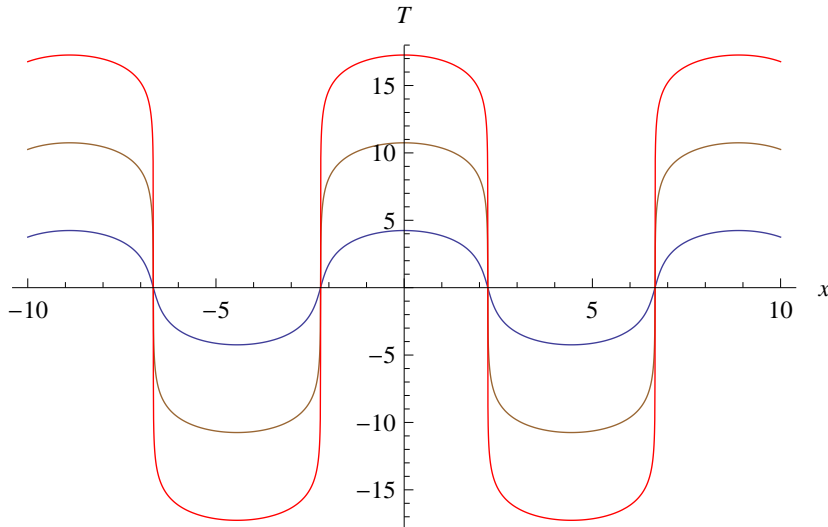


Figure 1: The euclidean rolling tachyon profile (2.6) is plotted here in (x, T) space for $\alpha = \frac{1}{\sqrt{2}}$ and three different values of the free constant A . $A = 10^2$ for the blue curve, $A = 10^3$ for the brown curve and $A = 10^5$ for the red curve.

An interesting inhomogeneous solution of the TDBI equations of motion (in flat space and tachyon potential (1.1c)) is described by the equation

$$\sinh(\alpha T) = A \cos(\alpha x) . \quad (2.6)$$

This solution is a Euclidean version of the rolling tachyon solution [22]. x is one of the physical space coordinates and α the constant that appears in the tachyon potential (1.1c). A is a free real constant, *i.e.* a modulus of the solution.

For $A = 0$ we recover a non-BPS brane oriented along x . For finite A the open string tachyon of the non-BPS brane has condensed. On the worldsheet a marginal boundary interaction of the form

$$\delta \mathcal{L}_{ws} = \mu_B \cos(\alpha x) \quad (2.7)$$

has been turned on. As we increase the magnitude of A the profile of the solution develops larger and larger regions oriented along the T direction (see Fig. 1). Hence, for $A = \pm\infty$ we recover an array of BPS-antiBPS branes separated by a distance

$$L_* = \frac{\pi}{\alpha} \quad (2.8)$$

controlled by the TDBI parameter α . For the type II string theory value $\alpha = \frac{1}{\sqrt{2}}$ the separation is $L_* = \pi\sqrt{2}$. This particular value of L has a special significance from the point of view of the $D\bar{D}$ system. When the D and \bar{D} -branes are separated by this distance the

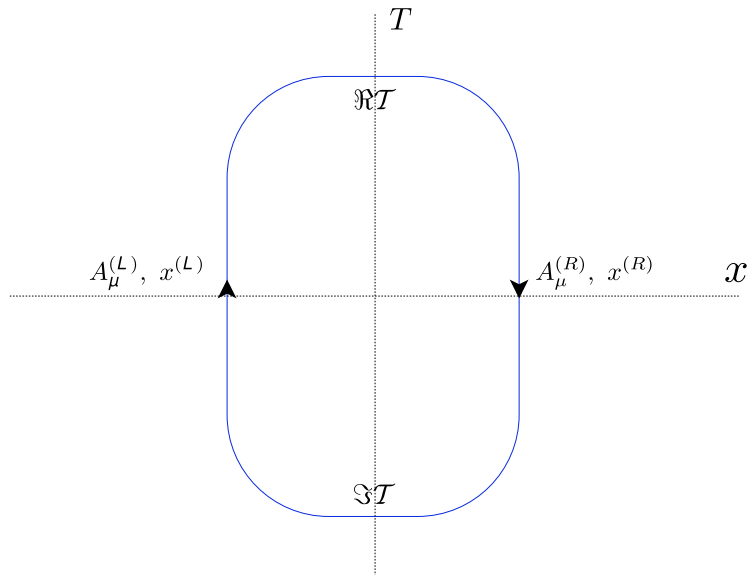


Figure 2: *The tachyon paperclip profile. Different parts of its geometry capture different degrees of freedom of the $D\overline{D}$ system.*

complex $D\overline{D}$ tachyon \mathcal{T} is massless. This fact is expressed naturally in the TDBI solution by the marginality of the parameter A . The TDBI action (1.1) recovers in this way a critical property of the open string theory of the $D\overline{D}$ system.

Reversing the order of this observation it is also instructive to make the following point. Assume that we knew that the TDBI action (1.1a) is a good description of open string dynamics for non-BPS branes, but we had no prior knowledge of the precise form of the tachyon potential $V(T)$. A quick way to determine the potential would have been to demand that the action

$$\mathcal{S} = - \int dx V(T) \sqrt{1 + \left(\frac{dT}{dx}\right)^2} \quad (2.9)$$

has a one-parameter family of periodic solutions with a modulus-*independent* period (as in Fig. 1). One can easily verify that the only potential with this property is the $1/\cosh$ potential.

The above discussion demonstrates that the abelian TDBI action has the necessary ingredients to provide an effective description not only of the BPS and non-BPS branes, but also of the $D\overline{D}$ system. For the latter it is more appropriate to consider a *closed* curve in (x, T) space with a ‘paperclip’ profile of the form depicted in Fig. 2. To first approximation we may view this profile as arising from the joining of a large positive- A half-period of the solution (2.6) (upper part) with a large negative- A half-period of the same solution (lower

part). When the T -width of the paperclip profile is large, the legs —perpendicular to the x direction— represent a D-brane separated by a distance L_* from a $\overline{\text{D}}$ -brane. The low-energy degrees of freedom on each of these legs are an abelian gauge field, $A_a^{(L)}$ or $A_a^{(R)}$, and the corresponding transverse scalars, $x^{(L)}$ or $x^{(R)}$. The upper and lower parts of the paperclip, associated with the A mode of the TDBI solution, capture the real and imaginary parts of the complex $\text{D}\overline{\text{D}}$ tachyon \mathcal{T} . Ref. [14] demonstrates explicitly how one recovers the full set of $\text{D}\overline{\text{D}}$ modes from the single abelian gauge field A_a and real scalar T of the abelian TDBI description.

A crucial part of this dictionary is a non-trivial transformation relating the $\text{D}\overline{\text{D}}$ \mathcal{T} and the TDBI T . For $T \gg 1$ and \mathcal{T} real this transformation takes the form

$$\mathcal{T} \sim A^{-1} \sim e^{-\alpha T} \quad (2.10)$$

where T is evaluated at its maximum on the central vertical axis of the paperclip. The inversive nature of this relation expresses the anticipated feature that \mathcal{T} is mildly condensed when the maximum value of T is large. Moreover, treating the paperclip amplitude A as a slowly varying field one recovers an action of the form

$$\mathcal{S} = -\frac{\tau_{p-1}^{\text{BPS}}}{2} \int d^p x \partial_\mu \phi \partial^\mu \phi \sum_{n=0}^{\infty} a_n \left(\frac{\partial_\mu \phi \partial^\mu \phi}{\phi^2} \right)^n \quad (2.11)$$

with calculable constants a_n and $\phi \sim A^{-1/2}$ an appropriately normalized field. This expansion suggests that the TDBI description is better suited to large values of $\mathcal{T} \sim \phi^2$.

In this language, the real time process of $\text{D}\overline{\text{D}}$ tachyon condensation takes a simple and geometric form. When the separation L of the paperclip legs is larger than the critical separation L_* (see eq. (2.8)), it is energetically favored for the paperclip to grow indefinitely in the T -direction leaving behind a $\text{D}\overline{\text{D}}$ system with $\mathcal{T} = 0$. This property captures the fact that the $\text{D}\overline{\text{D}}$ tachyon is massive in this case. In the opposite regime, $L < L_*$, it is energetically favored for the paperclip to shrink. There is a time-dependent solution that describes the tachyon-paperclip shrinking down to zero size (see Fig. 3). In this process the legs of the paperclip move closer and closer. In this manner, the time-dependent real tachyon T captures efficiently the complicated coupled dynamics of the complex $\text{D}\overline{\text{D}}$ tachyon \mathcal{T} and the transverse scalars $x^{(L)}, x^{(R)}$. The non-local nature of \mathcal{T} is incorporated naturally in this description.

In what follows we will argue that there is a similar effective field theory description of the open string dynamics on hairpin-branes in holographic backgrounds. Instead of a time-dependent tachyon-paperclip, in this case we have to consider a tachyon-paperclip ‘condensing’ along the radial direction of the holographic background.

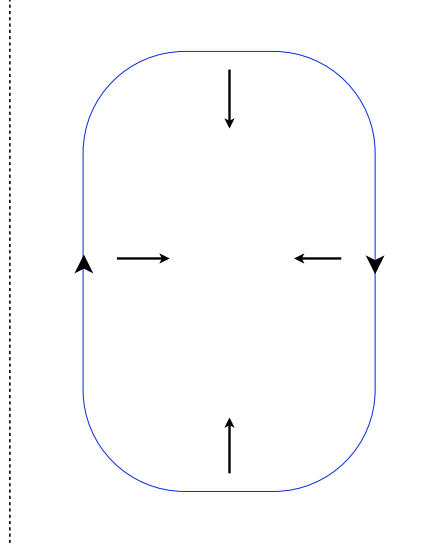


Figure 3: *Condensation of the $D\overline{D}$ tachyon in the TDBI language. For $L < L_*$ the tachyon-paperclip acquires a time-dependent evolution during which it shinks down to zero size.*

3. Hairpin-branes in holographic backgrounds

We will concentrate on two specific examples of holography: the Sakai-Sugimoto model for QCD and a closely related example of NS5-branes that describes a non-supersymmetric version of the six-dimensional Little String Theory compactified on a circle. In this section we review the main features of these models (emphasizing common properties and differences) and set up our notation.

3.1. Lightning review of the Sakai-Sugimoto model

The Sakai-Sugimoto model [3, 4] is based on the following configuration of D4 and D8-branes in type IIA string theory

$$\begin{array}{cccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 D4 : & \times & \times & \times & \times & \bullet & & & & & \\
 D8 : & \times & \times & \times & \times & & \times & \times & \times & \times & \times \\
 \overline{D8} : & \times & \times & \times & \times & & \times & \times & \times & \times & \times
 \end{array} \tag{3.1}$$

The worldvolume theory on the N_c D4-branes gives rise to a maximally supersymmetric $U(N_c)$ gauge theory in five dimensions. The x^4 direction is compactified (hence the solid circle in the D4 line of (3.1)) with supersymmetry breaking antiperiodic boundary conditions

for the fermions. In the deep infrared (IR), below the Kaluza-Klein (KK) scale set by the radius R_4 , the dynamics is dominated by the four-dimensional Yang-Mills theory with gauge group $SU(N_c)$. The intersection of the D4-branes with N_f D8-branes supports chiral fermions in the fundamental representation of the gauge group. These fermions propagate in the 3+1 dimensions common to both sets of branes. Similarly, the intersection with N_f $\overline{\text{D8}}$ -branes provides an analogous set of anti-chiral fermions. The D8 and $\overline{\text{D8}}$ stacks are separated by a distance L in the x^4 direction.

At weak 't Hooft coupling, the IR dynamics of this system is captured by a non-local version of the Nambu-Jona-Lasinio model [23, 6]. In this paper we are more interested in the strong 't Hooft coupling regime. In the probe approximation, where $N_f/N_c \ll 1$, this regime is captured holographically by N_f D8-branes in the supergravity background of the Wick-rotated black D4-brane

$$ds^2 = \left(\frac{u}{R}\right)^{\frac{3}{2}} \left(-dt^2 + (dx^i)^2 + f(u)(dx^4)^2\right) + \left(\frac{R}{u}\right)^{\frac{3}{2}} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \quad (3.2a)$$

$$e^\Phi = g_s \left(\frac{u}{R}\right)^{\frac{3}{4}}, \quad F_4 = 3\pi N_c \ell_s^3 \varepsilon_4, \quad (3.2b)$$

$$f(u) = 1 - \frac{u_{\text{KK}}^3}{u^3}. \quad (3.2c)$$

This solution is completely fixed by three numbers: $\frac{u_{\text{KK}}}{\ell_s}$, $\frac{R}{\ell_s}$ and N_c . The absence of a conical singularity at $u = u_{\text{KK}}$ fixes the radius R_4 of x^4 to the value

$$R_4 = \frac{2}{3} \frac{R^{\frac{3}{2}}}{u_{\text{KK}}^{\frac{1}{2}}}. \quad (3.3)$$

g_s is related to $\frac{R}{\ell_s}$ and N_c through the relation

$$g_s = \frac{1}{\pi N_c} \left(\frac{R}{\ell_s}\right)^3. \quad (3.4)$$

At strong 't Hooft coupling curvatures are small everywhere, but the string coupling e^Φ becomes large in the asymptotic region $u \gg 1$. For that reason, one usually sets a strong coupling cutoff at

$$u_{\text{max}} \sim g_s^{-4/3} R. \quad (3.5)$$

The D8 and anti-D8 branes reconnect in this background and form N_f hairpin-like branes in the (u, x^4) part of the geometry which has a cigar-like topology. Using the DBI action one finds a one-parameter family of solutions parameterized by the turning point value u_0 . At $u \gg u_0$ the two branches of the D8-brane are separated by a distance L which is a function

of u_0 . As we increase u_0 , $L(u_0)$ decreases. When $u_0 = u_{\text{KK}}$, *i.e.* when the brane reaches the tip of the (u, x^4) cigar, the asymptotic separation L becomes maximal and the D8-brane is situated at antipodal points of the x^4 circle.

The geometric reconnection of the D8-branes is a nice feature of the DBI solution that expresses naturally the flavor chiral symmetry breaking that takes place in the dual gauge theory. Nevertheless, it is clear already on the basis of the general holographic dictionary that the DBI description cannot be the full story. In this description the order parameter of chiral symmetry breaking is absent and with it an important set of open string degrees of freedom that affect the low-energy physics. For that reason a more appropriate description of the open string dynamics of the D8-branes is needed. We will return to propose such a description in section 5.

3.2. A Little String Theory analog of the Sakai-Sugimoto model

Before addressing the dynamics of hairpin-branes in the Sakai-Sugimoto model it is instructive to consider a closely related situation where instead of N_c D4-branes we have k NS5-branes. Since there are only NSNS fluxes in the background geometry of this system an explicit worldsheet description of both closed and open string dynamics is possible with the use of the RNS formalism. With this technical advantage we can use this system to set the standard lore: identify the crucial properties of string theory and test any proposals for an effective field theory that aspires to reproduce them.

Specifically, consider the following brane configuration in type IIB string theory

$$\begin{array}{cccccccccc}
& 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
NS5 : & \times & \times & \times & \times & \bullet & \times & & & & \\
D1 : & \times & & & & & & + & & & \\
\overline{D1} : & \times & & & & & & + & & &
\end{array} \tag{3.6}$$

The worldvolume theory on $k \geq 2$ coincident NS5-branes gives rise at low energies to a six-dimensional Little String Theory [5], a very interesting interacting non-gravitational theory whose dynamics remains largely unknown. Once again we compactify the x^4 direction (one of the worldvolume directions of the NS5-branes) on a circle with radius R_4 imposing supersymmetry breaking antiperiodic boundary conditions for the fermions. A set of D1 and anti-D1 branes extend along the upper half of the x^6 direction (hence the $+$ in the second and third lines of (3.6)) and intersect the NS5-branes at $x^6 = 0$. The D1-branes are separated from the anti-D1-branes by a distance L along x^4 . T-dualizing along the x^1, x^2, x^3

or x^5 directions we can trivially obtain Dp and anti- Dp branes intersecting the NS5-branes in type IIA or type IIB string theory. In order to be concrete we will restrict our discussion to D1-branes oriented as in (3.6).

Before compactification the configuration of the NS5-branes is supersymmetric and the near-horizon geometry of k NS5-branes is given by the CHS background [24]

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + k(d\rho^2 + d\Omega_3^2) , \quad (3.7a)$$

$$\Phi = -\rho , \quad H_3 = dB_2 = k \varepsilon_3 \quad (3.7b)$$

where ε_3 is the volume form of the transverse three-sphere. String theory on this background is described by an exact CFT on the worldsheet. String propagation in the four-plane (6789) transverse to the NS5-brane worldvolume is described by the $\mathcal{N} = (1, 1)$ supersymmetric linear dilaton CFT and the $SU(2)_k$ WZW model. The core of this throat, at large negative ρ , has a strong coupling singularity.

In this background the D1-branes are aligned along the linear dilaton direction ρ and are pointlike in the transverse sphere. The separation L of the D1 and anti-D1-branes in the x^4 direction is arbitrary. In the NS- sector of the string stretching between a D1 and an anti-D1-brane there is a mode with mass squared [25, 26]

$$\mathbf{M}_{\mathcal{T}}^2 = \frac{L^2}{4\pi^2} - \frac{k}{4} . \quad (3.8)$$

This mode becomes massless when the separation L takes the critical value

$$L_* = \pi\sqrt{k} . \quad (3.9)$$

We observe that the mass formula (3.8) receives a negative contribution proportional to k , which is potentially large when k is large. The origin of this contribution lies at the non-trivial dependence of the wavefunction along the linear dilaton direction. As a result, this particular mode can be massless even for separations L_* much larger than the string scale, when $k \gg 1$. The mode we are currently describing is special for the following reason. For $L > L_*$ there are no tachyonic modes in the open string theory of the D1- $\overline{\text{D1}}$ system. As we lower L below L_* the above mode is the first one to become tachyonic.

After the compactification with supersymmetry breaking boundary conditions, the background of the NS5-branes becomes that of the Wick-rotated black NS5-brane. In the near-horizon limit

$$ds^2 = -dt^2 + \sum_{i=1, i \neq 4}^5 (dx^i)^2 + k d\Omega_3^2 + k \left(d\rho^2 + \tanh^2 \rho d\theta^2 \right) \quad (3.10a)$$

$$e^\Phi = \frac{g_s}{\cosh \rho} , \quad H_3 = k \varepsilon_3 . \quad (3.10b)$$

ρ is now a positive real number and $\theta = \frac{x^4}{\sqrt{k}}$. To avoid a singularity at $\rho = 0$, θ needs to be compactified, *i.e.* $\theta \sim \theta + 2\pi$. With $g_s \ll 1$, string theory on this background is everywhere perturbative and one can apply perturbative string theory techniques to study its properties. String propagation on the cigar geometry parametrized by (ρ, θ) is described on the worldsheet by an exact CFT, the $\mathcal{N} = (2, 2)$ $SL(2)_k/U(1)$ gauged WZW model [27]. Curvatures are small compared to the string scale when $k \gg 1$, but since we have an explicit string theory description of this system we can discuss its properties for any $k \geq 2$.

As in the case of the Sakai-Sugimoto model, the $D1\text{-}\overline{D1}$ pairs reconnect to form hairpin D1-branes that extend along the radial direction of the two-dimensional cigar and the $U(1) \times U(1)$ symmetry, which is local on the D1-branes, is broken to a diagonal $U(1)$. There is a corresponding two-parameter family of hairpin-brane solutions of the DBI action described by the embedding equation [28]

$$\theta(\rho) = \theta_0 + \arcsin \left(\frac{\sinh \rho_0}{\sinh \rho} \right) . \quad (3.11)$$

ρ_0 is the turning point of the brane and θ_0 is a trivial shift of the asymptotic angular position of the hairpin branches. The asymptotic separation L is ρ_0 -independent and takes automatically the critical value L_* that we encountered in eq. (3.9). According to the mass formula (3.8),⁵ there is a half-winding mode of the $D\overline{D}$ tachyon which is massless. This bifundamental mode, whose vev breaks the $U(1) \times U(1)$ symmetry, plays an important role in the open string dynamics of these branes as we proceed to review in the next subsection.

An interesting and consequential difference with the Sakai-Sugimoto case, which is worth highlighting here, can be traced to the DBI fact of modulus-dependence (in the Sakai-Sugimoto model) or modulus-independence (in the NS5 model) of the asymptotic separation L . Apart from this detail, gross features of hairpin-brane open string dynamics are expected to be common in these systems.

3.3. Open string theory in the near-horizon region of NS5-branes

The D1 hairpin-branes that we have just described have an exact treatment in string theory as boundary states in the $SL(2)/U(1)$ CFT [30, 31, 32, 33, 34, 35]. Here we summarize their most pertinent properties.

⁵This formula remains valid in the non-supersymmetric compactified background, see for instance [29].

The axial coset $SL(2)_k/U(1)$ that describes string propagation on the cigar-shaped background

$$ds^2 = k \left(d\rho^2 + \tanh^2 \rho d\theta^2 \right) , \quad \Phi = \Phi_0 - \log \cosh \rho \quad (3.12)$$

is a Kazama-Suzuki model [36] with $\mathcal{N} = (2, 2)$ worldsheet supersymmetry and central charge $c = 3 + \frac{6}{k}$. By mirror symmetry it is equivalent to the $\mathcal{N} = 2$ Liouville theory [37, 38]. The spectrum of closed and open string modes is organized according to the representation theory of $SL(2)/U(1)$. The worldsheet primary fields are labeled by five quantum numbers: $j, m, \bar{m}, s, \bar{s}$ (bars denote right-moving sector quantities). We will denote the corresponding vertex operators as

$$\mathcal{V}_{j,m,\bar{m}}^{(s,\bar{s})} \quad (\text{or } \mathcal{V}_{j,m}^{(s)} \text{ for the chiral version}) . \quad (3.13)$$

The triplet (j, m, \bar{m}) is a triplet of $SL(2)$ quantum numbers. j is the $SL(2)$ spin and m, \bar{m} are directly related to the momentum and winding quantum numbers (n, w respectively) in the angular direction θ of the geometry

$$m = \frac{k w + n}{2} , \quad \bar{m} = \frac{k w - n}{2} . \quad (3.14)$$

In the mirror $\mathcal{N} = 2$ Liouville theory the roles of n and w are exchanged. The (s, \bar{s}) are worldsheet fermion numbers ($s = 0$ for the NS sector and $s = 1$ for the R sector). In $\mathcal{N} = 2$ Liouville language the chiral version of the above vertex operators reads

$$\mathcal{V}_{j,m}^{(s)} = e^{-j\rho + i(m+s)\theta + isH} \quad (3.15)$$

where H is a worldsheet boson that bosonizes a complex fermion. The scaling dimension of this operator is

$$\Delta_{j,m}^{(s)} = \frac{(m+s)^2 - j(j-1)}{k} + \frac{s^2}{2} . \quad (3.16)$$

Observe the possibly negative contribution of the Casimir $-j(j-1)$ to the scaling dimension (and hence the mass squared of the corresponding spacetime mode) for real values of the spin.

The allowed values of the spin j depend on the representation of $SL(2)$. In the closed string spectrum two representations appear: *continuous* representations with

$$j = \frac{1}{2} + ip , \quad p \in \mathbb{R} \quad (3.17)$$

and *discrete representations* with

$$\frac{1}{2} < j < \frac{k+2}{2} , \quad \pm m = j + \mathbb{Z}_{>0} . \quad (3.18)$$

The continuous representations provide delta-function normalizable modes. p , the imaginary part of j , can be viewed as momentum in the radial direction ρ of the background. The discrete representations provide normalizable modes supported near the tip of the cigar (these modes can be viewed as localized twisted-sector modes, to use a language familiar from orbifolds). What we want to capture eventually with an effective field theory is the dynamics of a mode of the $D\bar{D}$ tachyon that comes from such a discrete representation.

There is a feature of closed string theory in this background that we want to emphasize before proceeding to review the open string theories of interest. The background metric and dilaton (3.12), which solve the leading order (in α') supergravity equations, do not receive any α' corrections in perturbation theory and are exact to all orders in $1/k$ [39, 40]. From the point of view of the asymptotic cylinder, at $\rho \rightarrow \infty$, the cigar-shaped ‘bending’ of the geometry appears as a worldsheet interaction with vanishing momentum and winding of the form

$$\tilde{\mu}_{bulk} \int d^2 z (\partial H - k \partial \theta)(\bar{\partial} H - k \bar{\partial} \theta) e^{-\rho} . \quad (3.19)$$

This interaction appears as a D-term deformation in the $\mathcal{N} = (2, 2)$ worldsheet Lagrangian.

It is well known, however, that string theory on this background receives non-perturbative corrections in the form of a ‘winding tachyon condensate’. This condensate, which is none other than the winding $\mathcal{N} = 2$ Liouville interaction, is an F-term deformation of the form

$$\mu_{bulk} \int d^2 z e^{-\frac{k}{2}(\rho + i\tilde{\theta}) + i(H + \bar{H})} + \text{c.c.} . \quad (3.20)$$

It involves the vertex operators $\mathcal{V}_{\frac{k}{2}, \mp \frac{k+2}{2}, \mp \frac{k+2}{2}}^{(\pm 1, \pm 1)}$. $\tilde{\theta}$ is the T -dual of the angular coordinate θ . One can easily verify, using the general formula (3.16), that these vertex operators have scaling dimensions (1,1) and give rise to marginal interactions on the worldsheet, *i.e.* massless fields in spacetime. It should be appreciated that the strengths of these interactions ($\tilde{\mu}_{bulk}; \mu_{bulk}, \bar{\mu}_{bulk}$) are not independent parameters and that these interactions appear simultaneously in the worldsheet CFT.⁶ From the worldsheet point of view these interactions play the role of dual screening charges [41] (see also [42]). Their simultaneous presence is intimately related to the mirror symmetry equivalence between the $\mathcal{N} = 2$ cigar coset and the $\mathcal{N} = 2$ Liouville theory. Mirror symmetry converts the non-perturbative winding $\mathcal{N} = 2$ Liouville condensate of the cigar coset to the standard momentum tachyon condensate of the $\mathcal{N} = 2$ Liouville theory. We will now see that a similar picture of dual worldsheet interactions characterizes the D1 hairpin-branes of interest.

⁶The precise relation between these parameters is not important here. It can be found for example in [18].

In the worldsheet boundary CFT the D1 hairpin-branes can be formulated as boundary states $|J, M\rangle$ labeled by two parameters [33, 18]. M is a parameter related to θ_0 in (3.11) and has trivial physics. Here we will set it to zero. J takes the complex value $\frac{1}{2} + iP$. P , which is a non-negative real number, is related to the turning point ρ_0 of the DBI profile (3.11) (see eq. (4.46) below for a more precise version of this relation). In particular, $P = 0$ when $\rho_0 = 0$.

The parameters (J, M) control the strength of the boundary worldsheet interactions that characterize the open string theory on these branes. The details of these interactions in $\mathcal{N} = 2$ Liouville language can be found in Ref. [18] (see section 5.1 in that paper). Two kinds of boundary worldsheet interactions are simultaneously turned on. The first one is a holomorphic square root of the bulk interaction (3.19) proportional to

$$\tilde{\mu}(\partial H - k\partial\theta)e^{-\rho} . \quad (3.21)$$

The second one is a holomorphic square root of the bulk interaction (3.20) proportional to

$$\mu e^{-\frac{k}{2}(\rho+i\tilde{\theta})+iH} \quad (3.22)$$

together with its complex conjugate. In terms of the brane label P

$$\mu\bar{\mu} \sim \sinh(2\pi P) , \quad \tilde{\mu} \sim \sinh\left(\frac{2\pi P}{k}\right) . \quad (3.23)$$

Boundary fermions are needed to express accurately these interactions (see Ref. [18] and appendix A). Note that the first relation in (3.23) is only valid for non-vanishing P . In fact, μ and $\bar{\mu}$ are non-zero for $P = 0$. More details can be found in appendix A.

These interactions can be viewed as open string analogs of (3.19) and (3.20). The first interaction (3.21) has zero winding and captures the geometric bending of the D1-brane. As we will see, this is the piece of the open string theory captured quite well in the asymptotic region by the DBI action. The second interaction is non-geometric and involves a massless half-winding mode of the $D\bar{D}$ tachyon. This is the mode we need to incorporate in the effective field theory description of these branes extending the DBI action. The present explicit string theory example clarifies how this bifundamental mode appears in the open string spectrum and why it is relevant for open string dynamics. In particular, notice that none of the hairpin-branes has a vanishing condensate of this mode and none of these branes can be described fully by the DBI action.

To summarize the lessons of the string theory analysis, we conclude that the chiral symmetry breaking $U(1)_L \times U(1)_R \rightarrow U(1)_{diag}$ that one observes on the D1-branes geometrically

as a reconnection effect is not unrelated to the physics of the $D\overline{D}$ tachyon which is the right field to carry the order parameter for this breaking. String theory knows at a fundamental level about both of these aspects and incorporates them simultaneously. In order to capture them efficiently with an effective field theory description one has to write an action that generalizes the DBI action and incorporates the effects of the $D\overline{D}$ tachyon. We will see that this is naturally achieved by the abelian TDBI action outlined in section 2. We are going to claim that this picture is not specific to the example of NS5-branes and that a qualitatively similar picture applies also to other situations, *e.g.* the Sakai-Sugimoto model.

4. Effective field theory description of D-branes intersecting NS5-branes

This is the main section of this paper. Our primary task is to set up an effective field theory description that reproduces the exact open string theory features that were summarized above.

4.1. The TDBI action and its equations of motion

In order to set up an effective action for the D1 hairpin-brane in the background of NS5-branes we will start, following the idea of [14], from the tachyon-DBI action of a non-BPS D2-brane that wraps the two-dimensional cigar geometry (3.12).⁷ Applied to this background the TDBI action (2.1) reads (with vanishing gauge field)

$$\mathcal{S} = - \int d\rho d\theta \sinh \rho V(T) \sqrt{1 + \frac{1}{k}(\partial_\rho T)^2 + \frac{1}{k} \coth^2 \rho (\partial_\theta T)^2} . \quad (4.1)$$

Since we want to consider profiles of the tachyon that are not single-valued functions of θ it will be also convenient to choose a different parametrization

$$T = T(\rho, \sigma) , \quad \theta = \theta(\rho, \sigma) , \quad \sigma \in [0, 2\pi) . \quad (4.2)$$

In this parametrization the TDBI action becomes

$$\mathcal{S} = - \int d\rho d\sigma \sinh \rho V(T) \sqrt{(\partial_\sigma \theta)^2 + \frac{\coth^2 \rho}{k} (\partial_\sigma T)^2 + \frac{1}{k} (\partial_\rho \theta \partial_\sigma T - \partial_\rho T \partial_\sigma \theta)^2} . \quad (4.3)$$

The Euler-Lagrange equations of the action (4.1) can be written in the form

$$\partial_\rho \left(\frac{1}{k} \sinh \rho \frac{V}{\sqrt{W}} \partial_\rho T \partial_\theta T \right) = \partial_\theta \left(\sinh \rho \frac{V}{\sqrt{W}} \left(1 + \frac{1}{k} (\partial_\rho T)^2 \right) \right) \quad (4.4)$$

⁷The branes of interest are point-like in the remaining transverse directions, *e.g.* the transverse 3-sphere, so we can ignore them.

where

$$W \equiv 1 + \frac{1}{k}(\partial_\rho T)^2 + \frac{1}{k} \coth^2 \rho (\partial_\theta T)^2 . \quad (4.5)$$

Varying the action (4.3) with respect to T and θ we obtain one independent differential equation of the form

$$\partial_\rho \left(\sinh \rho \frac{V}{\sqrt{Q}} (\partial_\rho \theta \partial_\sigma T - \partial_\rho T \partial_\sigma \theta) \partial_\sigma T \right) = \partial_\sigma \left(\sinh \rho \frac{V}{\sqrt{Q}} (-k \partial_\sigma \theta + (\partial_\rho \theta \partial_\sigma T - \partial_\rho T \partial_\sigma \theta) \partial_\rho T) \right) \quad (4.6)$$

where

$$Q \equiv (\partial_\sigma \theta)^2 + \frac{\coth^2 \rho}{k} (\partial_\sigma T)^2 + \frac{1}{k} (\partial_\rho \theta \partial_\sigma T - \partial_\rho T \partial_\sigma \theta)^2 . \quad (4.7)$$

We are looking for solutions of the equations of motion of these actions with non-trivial profiles of $T = T(\rho, \theta)$ (or $T = T(\rho, \sigma)$, $\theta = \theta(\rho, \sigma)$) that behave at large ρ like the tachyon-paperclip profile of Fig. 2. Instead of condensing in time, the tachyon-paperclip will now condense in the radial direction of the holographic background. The elementary degrees of freedom of the D1 hairpin-brane, *e.g.* the transverse scalar and $D\overline{D}$ tachyon associated to the worldsheet boundary interactions (3.21) and (3.22) respectively, will emerge in this description along the lines of the discussion in section 2.

In (4.1) we left the tachyon potential $V(T)$ as an undetermined function. In flat space, a derivation from first principles [15, 16] fixed the potential to the $1/\cosh$ form (1.1c). Motivated by the universality of the form of the DBI action, it would seem natural to propose that there is no background dependence in the tachyon potential and that the $1/\cosh$ form should continue to apply in general curved backgrounds. Nevertheless, both in the derivation of [15, 16] and in the discussion of boundary string field theory [43, 44] the tachyon potential is closely related to the disc partition function of string theory which is a quantity that depends explicitly on the closed string background over which we are computing. Furthermore, in the computation of [15, 16] the disc partition function is calculated in the background of the rolling tachyon solution, whose specifics are also expected to be dependent on the closed string background. Since it is currently unclear how the background will affect the tachyon potential in these computations, we will adopt a strategy in which we treat $V(T)$ as a free function. Our goal is to find tachyon-paperclip solutions of the general TDBI equations that reproduce some of the key features of the exact string theory answer. In what follows we will achieve this goal with solutions that require the modification of the tachyon potential for generic k . At the same time, we observe that the TDBI equations with the $1/\cosh$ potential (1.1c) admit (in a specific regime) a solution with features that are not expected from string theory for $k > 2$. These results could be viewed as an indication that the $1/\cosh$ potential

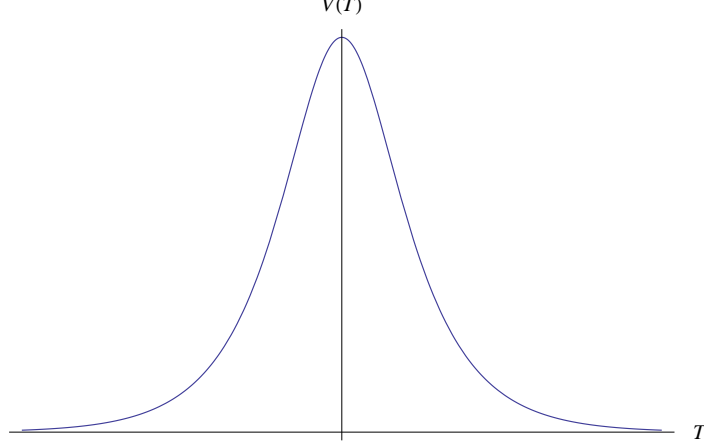


Figure 4: A graphical depiction of the general form of the tachyon potential $V(T)$.

is indeed modified in curved backgrounds, however, one would need a more comprehensive analytical control of the TDBI equations to make a conclusive argument on this issue. We do not claim to have a complete argument of this sort in this paper.

4.2. Constraining the tachyon potential: $|T| \ll 1$

With the above considerations in mind, we assume that the tachyon potential is an analytic function of the general form depicted in Fig. 4. It is symmetric under $T \rightarrow -T$, it has a maximum at $T = 0$, and it decreases monotonically with increasing $|T|$ towards zero for $|T| \rightarrow \infty$. A Taylor expansion of the potential around $T = 0$ gives to quadratic order

$$V(T) \simeq V(0) \left(1 - \frac{1}{2} \alpha^2 T^2 + \mathcal{O}(T^4) \right) \quad (4.8)$$

where we have set

$$\frac{1}{V(0)} \frac{d^2 V}{dT^2} \Big|_{T=0} = -\alpha^2 . \quad (4.9)$$

$V(0)$ expresses the tension of the non-BPS brane (here a non-BPS brane wrapping the cigar). α^2 is directly related to the mass \mathbf{M}_T of the real tachyon that resides on the non-BPS brane.

To obtain the precise relation between α^2 and \mathbf{M}_T we expand the TDBI action (4.1) up to quadratic order in T and ∂T

$$\mathcal{S} \simeq -V(0) \int d\rho d\theta \sinh \rho \left(1 + \frac{1}{2k} (\partial_\rho T)^2 + \frac{\coth^2 \rho}{2k} (\partial_\theta T)^2 - \frac{1}{2} \alpha^2 T^2 \right) + \dots . \quad (4.10)$$

Defining a renormalized tachyon

$$S = \sqrt{\cosh \rho} T \quad (4.11)$$

the equation of motion of the quadratic action (4.10) becomes

$$\square S - \mathbf{M}_T^2 S = 0 \quad (4.12)$$

with

$$\mathbf{M}_T^2 = -\alpha^2 + \frac{1}{k} \left(1 - \frac{3}{4} \tanh^2 \rho \right) . \quad (4.13)$$

At the asymptotic infinity ($\rho \rightarrow \infty$) we read off the tachyon mass squared

$$\mathbf{M}_T^2(\infty) = -\alpha^2 + \frac{1}{4k} . \quad (4.14)$$

In the open string theory of the non-BPS D2-brane the NS- sector mode of interest has vanishing angular momentum and winding ($n = w = 0$) and belongs to the continuous representation with radial momentum $p = 0$, *i.e.* $SL(2)$ spin $j = \frac{1}{2}$. The mass of this tachyonic mode can be read off the formula

$$\mathbf{M}_T^2 = \Delta_{\frac{1}{2},0}^{(0)} - \frac{1}{2} = -\frac{1}{2} + \frac{1}{4k} \quad (4.15)$$

where eq. (3.16) was used to determine the second equality. Matching this expression to the TDBI result (4.14) requires

$$\alpha = \frac{1}{\sqrt{2}} , \quad (4.16)$$

a k -independent value that matches the one encountered for non-BPS branes in type II string theory in flat space. Notice that the mass formula (4.15) reproduces correctly the flat space result as we send $k \rightarrow \infty$ and make the cigar geometry arbitrarily weakly curved.

4.3. Constraining the tachyon potential: $|T| \gg 1$ and paperclip asymptotics

In the previous subsection we have determined the leading order quadratic behavior of the tachyon potential around the open string vacuum. We will now discuss the behavior of the potential around the closed string vacuum at $|T| \gg 1$. Our main purpose is to find a sensible tachyon-paperclip solution of the TDBI equations at $\rho \rightarrow \infty$ and to match the ρ -dependence of this solution to the exact information (3.22) from open string theory. We will achieve this purpose with the assumption of a specific ansatz for the large- T behavior of the tachyon-paperclip solution. We will discover that this ansatz requires a modification of the tachyon potential (1.1c). The possibility of keeping the potential unchanged will be discussed briefly at the end of this subsection.

A motivated guess for the large- T behavior of the tachyon potential is

$$V(T) \sim e^{-\beta T} , \quad T \gg 1 . \quad (4.17)$$

For the potential (1.1c) $\beta = \alpha = \frac{1}{\sqrt{2}}$. In what follows, however, we will keep β as a free positive number that will be adjusted appropriately to satisfy the equations of motion. The negative T behavior of V is controlled by the symmetry $V(T) = V(-T)$. In order to match the exact information (3.22) from string theory we look for a large- T paperclip solution of the TDBI action with large- ρ asymptotics of the form

$$T \sim a + b\rho + c \log \cos \theta . \quad (4.18)$$

As a simple modification of the large- T behavior of the Wick-rotated version of the rolling tachyon solution in flat space (see section 2) this ansatz has automatically the qualitative features that we are after. The ρ -dependence has been chosen in a way that will soon allow us to reproduce the exact information (3.22) from string theory. We have also fixed the would-be legs of the tachyon-paperclip solution to lie at diametrically opposite points of the cigar S^1 , $\theta = \pm \frac{\pi}{2}$, as required from string theory. A trivial constant shift, which can be added freely to θ , will not be written out explicitly here. In this profile we want a to be an arbitrary constant that signals the presence of a marginal half-winding mode. The parameters b and c are k -dependent constants that need to be determined.

Inserting the ansatz (4.18) into the TDBI equations of motion (4.4) we obtain

$$\left[\beta - \frac{b}{k} (\beta b - 1) \left(1 + \frac{b^2}{k} \right)^{-1} \right] \left[\left(1 + \frac{b^2}{k} \right) \cos^2 \theta + \frac{c^2}{k} \sin^2 \theta \right] = \frac{c}{k} . \quad (4.19)$$

The constant a drops out as expected and remains free. This equation can be satisfied for general θ if and only if

$$c^2 = b^2 + k \quad (4.20)$$

in which case the θ -dependence disappears and we end up with the extra equation

$$\beta = \frac{1}{b + c} . \quad (4.21)$$

Once we fix from string theory the ρ -dependence of T , namely the constant b , we can determine the remaining constants, c and β , from eqs. (4.20) and (4.21).

We can read off the asymptotic ρ -dependence of the $D\bar{D}$ tachyon from the vertex operator \mathcal{V} (3.22). Defining an appropriately normalized version of this vertex operator, \mathcal{T} , we obtain

$$\mathcal{T} \equiv e^{d\rho} \mathcal{V} \sim \mu e^{-(\tilde{b}-d)\rho} + \dots + M e^{(\tilde{B}+d)\rho} + \dots \quad (4.22)$$

with

$$\tilde{b} = \frac{k}{2} , \quad \tilde{B} = \frac{k}{2} - 1 . \quad (4.23)$$

We are keeping explicit only the ρ -dependent piece of the vertex operator here and include both the normalizable branch with proportionality coefficient μ and the non-normalizable branch with coefficient M . Note that in the corresponding expansion of the $D\bar{D}$ tachyon in the Sakai-Sugimoto model, μ would express holographically the order parameter of chiral symmetry breaking and M would be related to the bare quark mass. The constant d that appears in the normalization factor $e^{d\rho}$ will be fixed appropriately in a moment.

In the asymptotic region, where $T \gg 1$, the exponential of T has a corresponding ρ -dependent expansion at $\theta = 0$

$$e^T \sim \mu_T e^{b\rho} + \dots + M_T e^{-B\rho} + \dots, \quad b \geq 0, \quad b + B > 0. \quad (4.24)$$

The first (leading) term in this expansion is captured already by eq. (4.18). The exponent of the first term of the subleading series, with proportionality constant M_T , can be determined from the TDBI equation of motion (4.4) using a general- k version of the differential equation (4.45) below (specifically, eq. (C.2) in appendix C). In terms of the leading exponent b we find

$$B = 1 - b + \frac{b^2}{k}. \quad (4.25)$$

The next task is to determine the relation between the two expansions (4.22) and (4.24). Using the results of appendix B we deduce that there is a non-trivial inversive transformation between T and \mathcal{T} of the form (2.10)

$$\mathcal{T} \sim e^{-\gamma T} \Leftrightarrow e^T \sim \mathcal{T}^{-\frac{1}{\gamma}}. \quad (4.26)$$

γ is a positive number related to the $U(1)_L - U(1)_R$ charge of e^T . We will soon see how γ is constrained.

At this point it is useful to recall the following mathematical identity

$$\frac{1}{\pi} \left(\frac{1}{|z|^2 e^\phi + e^{-\phi}} \right)^{2h} = \frac{1}{2h-1} e^{2(h-1)\phi} \delta^2(z) + \mathcal{O}(e^{2(h-2)\phi}) + \frac{e^{-2h\phi}}{\pi|z|^{4h}} + \mathcal{O}(e^{-2(h+1)\phi}). \quad (4.27)$$

The rhs of this equation is a large- ϕ expansion. This expansion appears, for instance, in the discussion of primary fields with spin $h-1$ in the Euclidean $SL(2, \mathbb{C})/SU(2)$ WZW model [45]. The subleading term $e^{-\phi}$ in the denominator of the lhs can play an important role when $|z|^2$ is sufficiently small. This effect gives rise to the first series in the rhs of (4.27).

Applying the identity (4.27) to the leading order terms of $\mathcal{T}^{-\frac{1}{\gamma}}$ we obtain

$$\begin{aligned} e^T \sim \mathcal{T}^{-\frac{1}{\gamma}} &\sim \left(\frac{1}{M e^{(\bar{B}+d)\rho} + \dots + \mu e^{-(\bar{b}-d)\rho} + \dots} \right)^{\frac{1}{\gamma}} \\ &\simeq \frac{\pi\gamma}{1-\gamma} \mu^{1-\frac{1}{\gamma}} e^{((\frac{1}{\gamma}-1)(\bar{b}-d)-(\bar{B}+d)\rho)} \delta(M) + \dots + M^{-\frac{1}{\gamma}} e^{-\frac{\bar{B}+d}{\gamma}\rho} + \dots \end{aligned} \quad (4.28)$$

Matching the leading terms of both branches in the expansions (4.28) and (4.24) we obtain the following relations

$$b = \left(\frac{1}{\gamma} - 1 \right) (\tilde{b} - d) - (\tilde{B} + d) , \quad (4.29a)$$

$$B = \frac{\tilde{B} + d}{\gamma} . \quad (4.29b)$$

Given \tilde{b} , \tilde{B} (see eq. (4.23)) and γ we can solve these equations (together with (4.25)) to obtain the unknown parameters b, c, d and β . In terms of γ we find

$$b = \sqrt{\frac{k}{\gamma}} \sqrt{k - 1 - k\gamma} , \quad \beta = \sqrt{\frac{\gamma}{k}} \frac{1}{\sqrt{k - 1 - k\gamma} + \sqrt{(k - 1)(1 - \gamma)}} , \quad (4.30a)$$

$$c^2 = k(k - 1) \frac{1 - \gamma}{\gamma} , \quad d = \frac{1}{2} \left(k + 2(1 - k)\gamma - \sqrt{k\gamma} \sqrt{k - 1 - k\gamma} \right) . \quad (4.30b)$$

The above expressions for b, β and d are real and finite if and only if

$$0 < \gamma \leq 1 - \frac{1}{k} . \quad (4.31)$$

The upper bound of this inequality is saturated when $T(\rho, 0)$ (4.18) asymptotes to a constant. In that case,

$$b = 0 , \quad c = \sqrt{k} , \quad d = 2 - \frac{1}{k} - \frac{k}{2} , \quad B = 1 , \quad \beta = \frac{1}{\sqrt{k}} \quad \text{and} \quad \gamma = 1 - \frac{1}{k} . \quad (4.32)$$

For these values and $k = 2$ the function $\cosh^{-1}(\frac{T}{\sqrt{2}})$ poses as a good candidate for the full tachyon potential $V(T)$. Indeed, this potential would reproduce correctly in this case both the small- T and large- T asymptotics, (4.8), (4.16) and (4.17), (4.32) respectively. We recall that this is also the tachyon potential for non-BPS branes in flat space in type II string theory (1.1c). In favor of this identification we note that similarities between the system of two NS5-branes and string theory in flat space have been observed before [46]. For generic $k > 2$, the $1/\cosh$ potential cannot reproduce both the small- T and large- T asymptotics that were postulated above.

It is tempting to speculate that the expressions (4.32) give the correct tachyon asymptotics for all values of k . Additional information is needed, however, to derive this result analytically. Strong numerical evidence in favor of this expectation is provided in appendix C.

One may also wonder whether it is possible to reproduce the available exact string theory information with more complicated tachyon-paperclip solutions and other tachyon potentials. For instance, we did not analyze the possibility of a more complicated tachyon-paperclip

solution that does not require the modification of the $1/\cosh$ potential (1.1c). Irrespective of the existence of such solutions, notice that in the particular case of (1.1c) the flat space solution (2.6) continues to solve the TDBI equations of motion to leading order at $\rho \rightarrow \infty$. For $\alpha = \frac{1}{\sqrt{2}}$ and $k > 2$ this fact seems to imply that there is a tachyon-paperclip solution that represents the asymptotic $D\bar{D}$ branches of a hairpin-brane with the wrong separation $L < \pi\sqrt{k}$. If this is true and the above solution can be properly extended to a regular ρ -dependent tachyon-paperclip solution beyond the large- ρ region it would appear to invalidate the action (4.1) with potential (1.1c) as a sensible description of D1 hairpin-branes for general $k > 2$; it would be a solution that represents a set of D1-branes that do not exist in the background of k NS5-branes.

4.4. Normalizable vs non-normalizable modes of the $D\bar{D}$ tachyon

The non-trivial transformation (4.28) has another interesting consequence. Comparing the expansions (4.24) and (4.28) we also learn that

$$\mu_T \sim \mu^{1-\frac{1}{\gamma}} \delta(M) \ , \quad M_T \sim M^{-\frac{1}{\gamma}} \ . \quad (4.33)$$

The roles of leading versus subleading branches are exchanged under the transformation (4.26). The coefficient μ that controls the subleading series in the expansion of the $D\bar{D}$ tachyon (4.22) controls in terms of T the coefficient of the leading series and vice versa for the coefficient M that controls the leading series in the expansion of the $D\bar{D}$ tachyon. In the language of holographic QCD, the branch associated to chiral symmetry breaking in the dual gauge theory becomes leading in the TDBI formulation. Adding a non-zero quark mass leads to a modification of the subleading branch that does not destroy the natural asymptotic boundary condition $T \rightarrow \infty$ (equivalently $\mathcal{T} \rightarrow 0$) at $\rho \rightarrow \infty$.

In previous discussions of tachyon dynamics in the context of the Sakai-Sugimoto model [8, 9] one encounters the issue of how to compromise this boundary condition with the presence of the leading branch that appears when we give bare mass to the quarks. The exponential divergence of the non-normalizable branch of the $D\bar{D}$ tachyon is simply incompatible with a vanishing \mathcal{T} at infinity and one has to resort into a delicate limiting prescription of how to make sense of a solution that incorporates holographically the bare quark mass [9].

Here we learn that this is a problem related more to the description rather than the fundamentals of the D-brane system. In the TDBI description both branches can be incorporated simultaneously and without any conflict with the natural asymptotic boundary conditions. Of course, if we insist to make the identification (4.28) strictly at infinity we

rediscover the issue in the form of a delta function contribution to the coefficient of the leading order branch (4.33). In holographic applications what matters is the precise dictionary between bulk and boundary quantities. It would be interesting to elaborate further on a more direct dictionary between TDBI variables and dual gauge theory quantities.

4.5. The solution near the turning point

So far we have discussed the asymptotic, $\rho \rightarrow \infty$, behavior of the paperclip solution, which is controlled to a large degree (*i.e.* sufficiently away from the paperclip legs) by the large- T properties of the tachyon potential. Now we want to discuss the behavior of the paperclip near the turning point of the hairpin-brane. This behavior is controlled by the small- T properties of the tachyon potential.

To capture the tachyon-paperclip behavior in this region we will use the parametrization (4.2) and solve the partial differential equation (4.6), (4.7) perturbatively around the turning point ρ_0 . We are looking for a capping-off solution with the properties

$$|T| \ll 1 , \quad (4.34a)$$

$$|\partial_\rho T|, |\partial_\rho \theta| \gg 1 , \quad (4.34b)$$

$$|\partial_\sigma T|, |\partial_\sigma \theta| \ll 1 , \quad (4.34c)$$

$$0 < \rho - \rho_0 \ll 1 . \quad (4.34d)$$

Expanding in powers of $\rho - \rho_0$ we set

$$T(\rho, \sigma) = \sqrt{\rho - \rho_0} \left(T_0(\sigma) + (\rho - \rho_0) T_1(\sigma) + \dots \right) , \quad \theta(\rho, \sigma) = \sqrt{\rho - \rho_0} \left(\theta_0(\sigma) + (\rho - \rho_0) \theta_1(\sigma) + \dots \right) . \quad (4.35)$$

We will see in a moment that the leading $\sqrt{\rho - \rho_0}$ behavior facilitates a natural solution for generic $\rho_0 \neq 0$. We insert this ansatz into the differential equation (4.6) and keep all terms up to the next-to-leading order in $\rho - \rho_0$. The expansion of the tachyon potential reads

$$V(T) = V(0) + \frac{1}{2} V''(0) T^2 + \mathcal{O}(T^4) = 1 - \frac{1}{2} \alpha^2 (\rho - \rho_0) T_0^2 + \mathcal{O}((\rho - \rho_0)^2) . \quad (4.36)$$

The leading zero-th order term of the differential equation (4.6) is automatically satisfied. At the next-to-leading order we find

$$2 \coth \rho_0 P_0 \partial_\sigma T_0 - k \partial_\sigma T_0 \frac{R_0}{P_0} + 2k \partial_\sigma^2 \theta_0 + \frac{k}{2} T_0 \frac{\partial_\sigma R_0}{P_0} = 0 \quad (4.37)$$

where

$$P_0 \equiv \frac{1}{2} \left(\theta_0 \partial_\sigma T_0 - T_0 \partial_\sigma \theta_0 \right) , \quad (4.38)$$

$$R_0 \equiv (\partial_\sigma \theta_0)^2 + \frac{\coth^2 \rho_0}{k} (\partial_\sigma T_0)^2 . \quad (4.39)$$

There are two interesting things happening here. First, the second order expansion of T and θ does not appear in the second order expansion of the equation of motion. Second, only the zero-th order (tension) term of the tachyon potential contributes. The tachyon mass term that depends on α drops out. Hence, the leading order behavior of the solution, expressed in terms of the functions T_0 and θ_0 , is independent of the details of the tachyon potential and acquires a more robust and universal character.

In order to solve the equation (4.37) we make the following ansatz

$$T = A\sqrt{\rho - \rho_0} \cos \sigma + \dots , \quad \theta = B\sqrt{\rho - \rho_0} \sin \sigma + \dots . \quad (4.40)$$

With this ansatz the σ -dependence drops out of the equation and what remains provides a polynomial relation between the constants A , B and $\tanh \rho_0$

$$2kB^2 \tanh^2 \rho_0 - A^2 B^2 \tanh \rho_0 + 2A^2 = 0 . \quad (4.41)$$

Hence, there are two parameters, say A and B , that control the solution. They map at the asymptotic infinity to the two independent coefficients μ_T and M_T (or μ and M) of the tachyon behavior. For a given pair of parameters (A, B) there are two turning points solving the quadratic equation (4.41)

$$\tanh \rho_0 = \frac{A}{4kB} \left(AB \pm \sqrt{A^2 B^2 - 16k} \right) . \quad (4.42)$$

The positivity of the discriminant requires $A^2 B^2 \geq 16k$. In general, when the discriminant is strictly positive, the two solutions are distinct and related formally by a simple parity transformation $AB \rightarrow -AB$ (or equivalently $\sigma \rightarrow -\sigma$). There is a single solution when the discriminant vanishes, *i.e.* when $A^2 B^2 = 16k$. The existence of two independent solutions for the turning point implies that the hairpin-brane is parametrized by two continuous parameters, A and B or μ and M , and an additional \mathbb{Z}_2 parameter. Since there is no known extra discrete parameter for vanishing M we conjecture that the standard hairpin-branes reviewed in section 3.3 are described by the vanishing discriminant case with

$$\tanh \rho_0 = \frac{A^2}{4k} , \quad B^2 = \frac{16k}{A^2} . \quad (4.43)$$

Another interesting feature of the relation (4.41) is the breakdown of the expansion (4.35), (4.40) as we take the limit $\rho_0 \rightarrow 0$. In that case, A is forced towards zero and B towards infinity making the leading order elliptical shape (4.40) highly asymmetrical along the θ -axis. This feature is reminiscent of the discontinuity that we observe in the boundary cosmological constants $\mu, \bar{\mu}$ in worldsheet CFT as we take $P \rightarrow 0$ (see appendix A).

4.6. Numerical results

Given the precise form of the tachyon potential one would like to solve the full equations (4.6), (4.7) and determine the entire profile of the condensing tachyon-paperclip solution. Since an analytic solution appears to be out of reach it would be desirable to explore a numerical evaluation of the solution. The optimal strategy would be to shoot from the region near the turning point, using the universal (*i.e.* tachyon potential independent) profile (4.40), towards the asymptotic infinity. We will not pursue this exercise in this paper. Instead, we will explore numerically some of the properties of the solution using an approximate tool that simplifies the differential equation to an equation for a single-variable function. This equation will play a similar role for the tachyon that the DBI equations play for the transverse scalars.

We will be setting boundary conditions at the asymptotic infinity. Estimates of the turning point will be provided by two independent sources: the singularity structure of the above approximate tool and the corresponding DBI solution. Already at this level of approximation we will recover a picture that corroborates the statements of the previous sections.

For illustration purposes we will fix the value of k in the rest of this subsection to $k = 2$ (see, however, appendix C for a numerical analysis of the case with general k). Although this is a highly stringy regime for the closed string background we anticipate that the TDBI action will continue, even in this case, to provide a sensible description. The numerical results verify this expectation. Our main motivation for considering this value of k is that the tachyon potential $\cosh^{-1}(\frac{T}{\sqrt{2}})$ poses as a good candidate in this case.

For starters, it is straightforward to check that the flat space solution (2.6), with $\alpha = \frac{1}{\sqrt{2}}$, is an exact leading order solution of (4.4) at the asymptotic infinity. Since the profile of this solution is essentially that of a square function for large values of the amplitude (see Fig. 1) this profile, with a ρ -dependent A , is expected to give a sensible estimate of the tachyon maximum $T(\rho, 0)$ as long as $T(\rho, 0)$ is large enough and the legs of the tachyon-paperclip are not converging fast on each other. Accordingly, we make the following ansatz for the tachyon

$$T(\rho, \theta) = \sqrt{2} \operatorname{arcsinh} \left(\cos \theta \sinh \frac{f(\rho)}{\sqrt{2}} \right) , \quad (4.44)$$

we insert this ansatz into the equation of motion (4.4), expand to leading order around $\theta = 0$

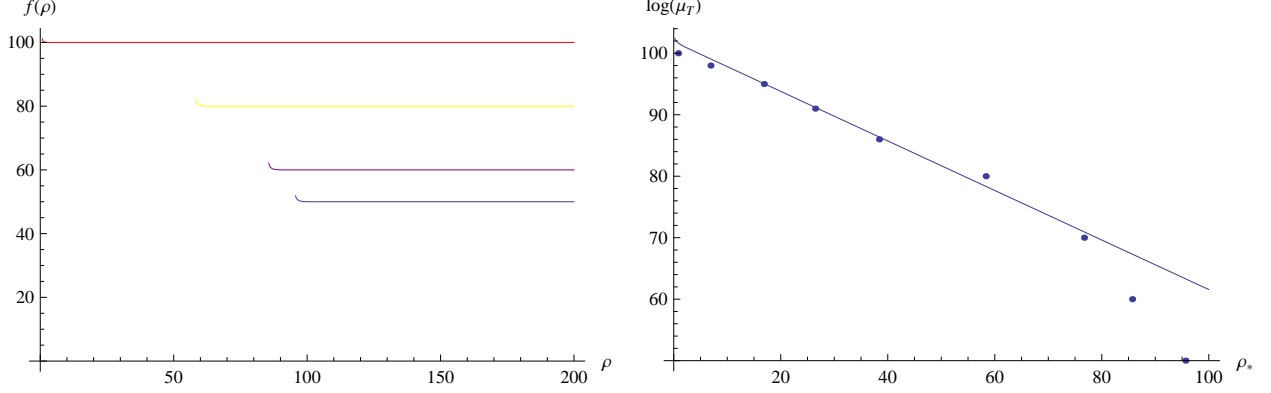


Figure 5: The left figure depicts the numerical evaluation of the differential equation (4.45) with four different boundary conditions set at $\rho = 100$. Each boundary condition is set by the coefficients μ_T and M_T in (4.24). All curves have the same $M_T = e^{146}$, a large value that implies via (4.33) $|M| \ll 1$. From the blue to the red curve we have $\mu_T = e^{50}, e^{60}, e^{80}, e^{100}$. The curves terminate on the left at the point ρ_* where the numerical evaluation encounters a singularity. The singularity occurs respectively at $\rho = 95.6352, 85.6667, 58.3069, 0.8018$ from the blue to the red curve. The nine plotted points in the right figure are based on a similar calculation with the same M_T and express $\log(\mu_T)$ as a function of the singularity point ρ_* . The solid blue curve is a fit obeying the equation (4.47), (4.49). In the vicinity of the tip of the cigar, $\rho_* \lesssim 1$, both the solid blue curve and the numerically determined curve $\log(\mu_T)$ exhibit a vertical rise to arbitrarily large values.

and obtain a differential equation for the function $f(\rho)$

$$\left(f' \cosh \rho - \frac{\sqrt{2}}{\sinh \rho} \tanh \left(\frac{f}{\sqrt{2}} \right) \right) (2 + f'^2) + 2 \sinh \rho f'' = 0 . \quad (4.45)$$

We claim that this function provides a useful first estimate of $T(\rho, 0)$ for a sufficiently large range of ρ .

We evaluated numerically the solution of the differential equation (4.45) for different boundary conditions with the use of MATHEMATICA. The result is plotted in Fig. 5. Keeping the asymptotic coefficient M_T (see eq. (4.24)) fixed and large we vary the coefficient μ_T . According to the transformation (4.28), this corresponds to keeping M fixed and small and varying μ . For each value of μ_T the numerical evaluation gives a function that remains essentially constant until it encounters a singularity at a point $\rho = \rho_*$ where the solution terminates. We will soon argue that ρ_* provides a good estimate of the turning point of the brane—an estimate that is comparable to the turning point predicted by the DBI solution.

As we increase μ_T , *i.e.* as we go from the blue to the red curve in the left plot of Fig. 5, the predicted turning point is moving closer and closer to the tip of the cigar geometry.

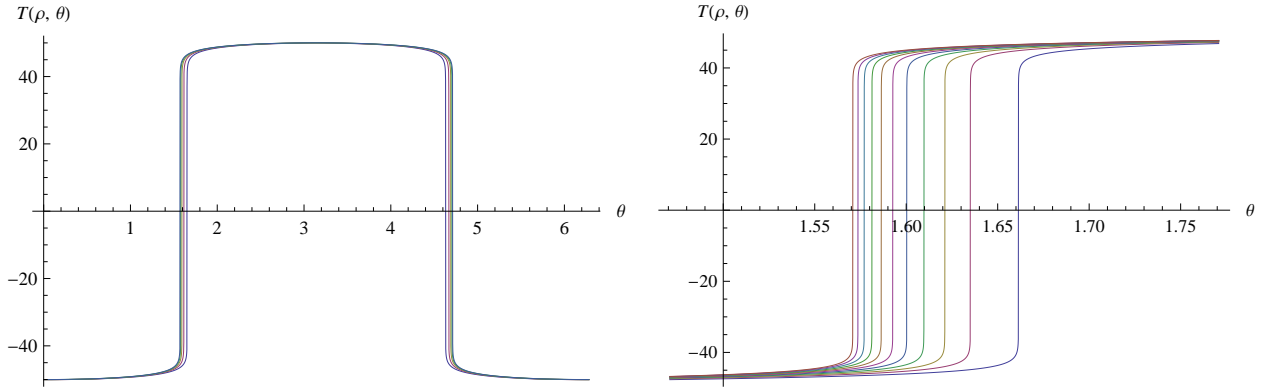


Figure 6: A plot of the tachyon profile $T(\rho, \theta)$ for several values of ρ ($\rho = 97.9, 98.2, 98.4, \dots, 99.8, 100$). Different values of ρ are represented by different colors. In this particular example we are solving numerically the full equation of motion (4.4) using the profile (4.44) as a boundary condition at $\rho = 100$ with radial derivative that gives $\mu_T = e^{50}$, $M_T = e^{146}$. The right figure provides a magnification of the solution near the position of one of the legs in the vicinity of $\theta = \pi/2$. We can see that with decreasing ρ the legs move towards the center at $\theta = \pi$.

This is precisely the behavior we expect to see. As we increase μ_T , μ decreases according to the first equation in (4.33). Smaller μ implies a turning point closer to the tip of the cigar as we can deduce from the first CFT relation in (3.23).

In CFT the turning point ρ_0 can be determined in terms of the brane label P in at least two different ways. One way is to define it implicitly by the mass of the ground state of an open string that stretches between a pointlike D0-brane at the tip of the cigar and the D1 hairpin-brane. Unfortunately, the spectrum of D0-D1 strings has not been computed explicitly in open string theory (such a computation involves the more complicated A-B type cylinder amplitudes, in the nomenclature of Ref. [35], which have not been studied sufficiently). However, the spectrum of D0-D2 strings, for D2-branes characterized by a similar label P as the D1-branes, has been studied extensively (see, for example, [35, 18]). For generic k , the mass squared of the ground state of a D0-D2 string contains the term $\frac{P^2}{k}$. A similar term is expected for the ground state of the D0-D1 string. Assuming this is correct, equating this term with the flat space tension term $\frac{k\rho_0^2}{4\pi^2}$ gives the following relation between P and ρ_0

$$P = \frac{k\rho_0}{2\pi} . \quad (4.46)$$

An alternative way to derive this relation, which produces an identical result, is based on a comparison of the semiclassical one-point functions of closed string vertex operators on the disc with the corresponding exact CFT one-point functions [30, 35].

Identifying ρ_* with the CFT ρ_0 and then combining the first equation in (4.33), the first equation in (3.23) (with $\mu = \bar{\mu}$) and the relation (4.46) we expect our numerical results to obey a relation of the form

$$\log \mu_T(\rho_*) = x - z \log \sinh(y\rho_*) , \quad (4.47)$$

where x, y, z are constants for which we anticipate analytically

$$y = k = 2 , \quad z = \frac{1}{2} \left(\frac{1}{\gamma} - 1 \right) = \frac{1}{2(k-1)} = 0.5 . \quad (4.48)$$

The precise value of x depends on the proportionality constant that appears in the first equation of (4.33). This constant depends on the precise coefficients of the relation (4.26) which we have left undetermined.

The right plot in Fig. 5 exhibits nine numerically determined values of the function $\log \mu_T(\rho_*)$. Amusingly, the function (4.47) provides a rather good fit of these data with fit values⁸

$$x = 101.507 , \quad y = 0.7941 , \quad z = 0.5075 . \quad (4.49)$$

The fit worsens as we approach the radius of the boundary conditions (in Fig. 5 this is $\rho = 100$). This is an expected deviation. The closer the turning point to the point of the boundary conditions the worse our approximation (4.44) becomes. In the opposite case, *i.e.* when the turning point is very close to the tip of the cigar, $\log(\mu_T)$ increases vertically in accordance with (4.47). Strictly at $\rho_* = 0$ (or $P = 0$) the first relation in (3.23) (and therefore also (4.47)) breaks down (see appendix A).

The fitted value of z , 0.5075, compares well with the analytic value 0.5. There is a greater mismatch for y . We expect $y = 2$ from eq. (4.46) and obtain 0.7941. At the same time, note that by focusing on ρ_* values in the rough range from $\rho_* \sim 10$ to $\rho_* \sim 80$, as we did in obtaining the fit values (4.49) (see footnote 8), we can approximate (4.47) by the linear relation

$$\log(\mu_T) = (x + z \log 2) - zy \rho_* . \quad (4.50)$$

In this range we are essentially fitting to determine a single parameter, the slope zy . The value anticipated by (4.48) is $zy = 1$. The value produced numerically by the fit is $zy \simeq 0.4$. We observe a rough numerical agreement which was not guaranteed to occur. We view this agreement, together with the qualitatively good behavior of the TDBI-derived curve

⁸This fitting has been performed with the `FindFit` command in `MATHEMATICA` using five numerically determined data points at $\log(\mu_T) = 70, 80, 86, 91, 95$.

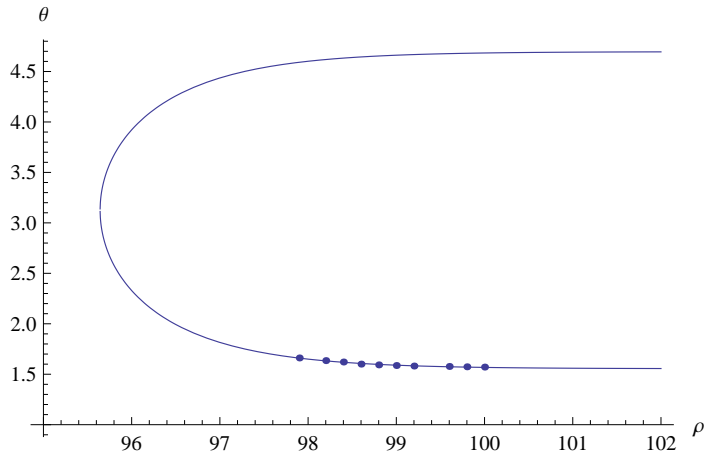


Figure 7: *Fitting the motion of the tachyon-paperclip legs with the DBI solution (3.11).*

$\log(\mu_T)(\rho_*)$ in the vicinity of the tip of the cigar and away from it, as non-trivial evidence for the consistency of our picture, the relevance of the approximate equation (4.45) and the fact that ρ_* provides a sensible estimate of the actual turning point of the brane. A similar picture arises at generic values of k (see appendix C for further details).

Further evidence for the approximate identification of ρ_* with the turning point of the brane is provided by comparing ρ_* to the turning point ρ_{DBI} predicted by the DBI solution. The bending described by the DBI solution is expected to deviate from the full TDBI result only within a small region around the turning point (see also the discussion in the following subsection 4.7). In that case, ρ_{DBI} is another, independent, estimate of the CFT turning point ρ_0 and should compare well with ρ_* . In Fig. 6 we have plotted the profile of $T(\rho, \theta)$ solving the full equation of motion (4.4) with boundary conditions at $\rho = 100$ corresponding to $\mu_T = e^{50}$, $M_T = e^{146}$.⁹ The f -estimate for the turning point of this brane was $\rho_* = 95.6352$. In Fig. 7 we plot ten numerically determined positions of the paperclip legs, *i.e.* ten values of θ for which $T(\rho, \theta) = 0$, as a function of ρ . We fit these values with the DBI solution (3.11) to obtain

$$\rho_{\text{DBI}} = 95.6425 \tag{4.51}$$

a value that indeed compares well with $\rho_* = 95.6352$.

These numerical results suggest that the ‘condensing’ tachyon-paperclip solution has a cigar-like shape in (T, ρ, θ) space that exhibits large radial gradients only near the turning

⁹The use of such a low value for μ_T is not ideal given the deviations of the fit in the right plot of Fig. 5. Our choice here is dictated purely by practical reasons related to the numeric evaluation of the solution of the full equation of motion (4.4).

point region. As we change the asymptotic coefficients μ , M , this shape changes and capping-off occurs at different radial distances.

4.7. *Concluding comments on the role of the DBI action*

We conclude this section with a few general comments on the role of the DBI action. We argued that the full low-energy dynamics of hairpin-branes in the background of NS5-branes cannot be described fully by the DBI action. We verified explicitly in worldsheet conformal field theory that these branes always have a non-vanishing condensate of a half-winding scalar mode that comes from the ground state of the NS– sector. Irrespective of the asymptotic separation of the hairpin legs, which is controlled by the number k of NS5-branes, this mode is always massless. Its mass squared receives large negative contributions from the non-trivial dependence of the wavefunction in the radial direction of the background. A consistent treatment of the dynamics of this field requires a different effective field theory treatment. We argued along the lines of Ref. [14] that the abelian tachyon-DBI action for non-BPS branes provides an effective field theory description that captures correctly the main features of open string dynamics of a single hairpin-brane.

Nevertheless, we can still use the DBI action as a partial description of the system that captures sufficiently well the dynamics of the transverse scalars (*i.e.* the geometric bending of the brane) in the asymptotic region where the legs of the tachyon-paperclip are long in the T -direction (see, for example, the fitting in Fig. 7.) Deviations from the DBI result occur more prominently in a region near the turning point where the tachyon-paperclip shrinks down to zero size. To determine these deviations for general μ and M one needs to employ the full TDBI action. Moreover, we have seen that the tachyon-paperclip takes an elliptical shape near the turning point and that already at the classical level even the notion of a well-defined transverse space position for the brane is lost in this region. This is in accordance with the fact that the $D\bar{D}$ tachyon is nearing there the closed string vacuum. The effective smearing of the brane near the turning point and the deviation from the DBI result are features that were also emphasized in Ref. [19].

It is interesting to note that the turning point region is also a delicate region for the DBI action itself. The DBI solution (3.11)

$$\theta(\rho) = \theta_0 + \arcsin\left(\frac{\sinh \rho_0}{\sinh \rho}\right) \quad (4.52)$$

develops large gradients near the turning point. For example, the second derivative

$$\frac{d^2\theta}{d\rho^2} = \frac{1}{2}\sqrt{\frac{\coth(\rho_0)}{2}}(\rho - \rho_0)^{-\frac{3}{2}} + \mathcal{O}((\rho - \rho_0)^{-1/2}) \quad (4.53)$$

diverges at $\rho \sim \rho_0$ and large gradients develop quickly in a region around the turning point.

We conclude by highlighting some of the implications of the intimate relation between the dynamics of the transverse scalars and the $D\overline{D}$ tachyon. This relation can be used to extract useful indirect information about the full dynamics of the system from the properties of the DBI solution. In the TDBI formulation, where this relation is manifest geometrically, one is instructed to think in terms of a single object —the tachyon-paperclip— which incorporates both degrees of freedom. This point of view implies, in particular, that a modulus of the DBI solution, *e.g.* the turning point value ρ_0 in our present example, is necessarily a modulus of the full tachyon-paperclip and therefore also a modulus of the $D\overline{D}$ tachyon. On a practical level, this is a way in which the DBI action can be used to read off quickly some of the properties of the $D\overline{D}$ tachyon, *e.g.* that there is a marginal tachyon condensate on the disc worldsheet with a specific winding quantum number. As another implication of this relation, the flavor chiral symmetry breaking that occurs geometrically by reconnection in the DBI solution is not independent of the $D\overline{D}$ tachyon condensation which carries explicitly the order parameter for this breaking.

5. Flavor D8-branes in the Sakai-Sugimoto model

Many of the qualitative features of flavor branes in the $NS5-Dp-\overline{Dp}$ system are expected to persist also in the case of the Sakai-Sugimoto model, *i.e.* when we replace the NS5-branes with D4-branes. More specifically, we still expect the open string theory on the D8-branes to be controlled by two closely related interactions: an interaction with non-vanishing winding of the $D\overline{D}$ tachyon and an interaction with vanishing winding of the transverse scalars. An additional argument in favor of this picture was given in [6]. Both interactions are relevant for the low-energy dynamics of the brane irrespective of how large the asymptotic separation of the D8 hairpin legs is. Only the latter interaction with vanishing winding will be captured efficiently by the DBI action and only in a region of the solution where the $D\overline{D}$ tachyon is mildly condensed. Marginal directions in the space of solutions of the DBI action imply marginal directions in the coupled system of transverse scalars and $D\overline{D}$ tachyon.

To determine the low-energy dynamics of the flavor D8-branes in the Sakai-Sugimoto model we need an effective action that incorporates the interactions of the $D\overline{D}$ tachyon.

Extending the lessons of the previous sections we would like to propose that the tachyon-DBI action for a non-BPS D9-brane is a promising tool for this purpose. The equilibrium configurations of the D8-branes can be determined by solving the equations of motion of the tachyon-DBI action

$$\mathcal{S} = - \int du dx^4 u^{13/4} V(T) \sqrt{1 + g(u) (\partial_u T)^2 + \frac{1}{g(u)} (\partial_{x^4} T)^2}, \quad g(u) = \left(1 - \frac{u_{\text{KK}}^3}{u^3}\right) \left(\frac{u}{R}\right)^{\frac{3}{2}}, \quad (5.1)$$

or the equations of motion of the corresponding action in the (ρ, σ) parametrization of T and x^4 , to find ‘condensing tachyon-paperclip’ solutions with the right asymptotics. In the language of the complex-valued $\text{D}\overline{\text{D}}$ tachyon \mathcal{T} the asymptotics are controlled by two parameters: the parameter of the normalizable branch (with strength μ) and the parameter of the non-normalizable branch (with strength M). Holographically, μ controls the order parameter of flavor chiral symmetry breaking and M the bare quark mass. There is a non-trivial transformation that relates \mathcal{T} with T and μ, M with the coefficients of the leading and subleading branches of $T(u, 0)$ evaluated at the maximum or minimum of the tachyon-paperclip. This transformation is part of the holographic dictionary when expressed in the language of the tachyon-DBI action.

In previous sections we kept the tachyon potential $V(T)$ free. Once again, we would like to know what kind of potentials reproduce sensible solutions. At large- T an exponential behavior of the form (4.17)

$$V(T) \sim e^{-\beta T}, \quad T \gg 1, \quad (5.2)$$

which is appropriate for the NS5-brane case, has the following feature in the Sakai-Sugimoto model. It is simple to verify that in this case the equation of motion following from the variation of the action (5.1) depends only on the tachyon derivatives $\partial_u T$ and $\partial_{x^4} T$. Hence, a tachyon solution can be shifted freely by a constant. In periodic solutions of the general type of Fig. 1 this free shift occurs without changing the period in the angular x^4 direction. If it is not possible to fix this free shift by some extra condition, *e.g.* a regularity condition, this freedom would imply that there is a one-parameter family of tachyon-paperclip solutions with a modulus-independent period as in the NS5-brane case. This feature would contradict the information coming from the DBI action, where one finds a one-parameter family of hairpin solutions with a modulus-dependent asymptotic separation. In that case, one would hope to put further constraints on the precise form of the tachyon potential $V(T)$ by making further use of the linked properties of the $\text{D}\overline{\text{D}}$ tachyon and transverse scalars.

The tachyon-DBI formulation also changes the way we compute mesonic spectra in holo-

graphic setups. Typically, in order to determine the structure of the mesonic spectra, we compute fluctuations of the transverse scalars and gauge fields using the DBI action. In the Sakai-Sugimoto model we need to add the $D\bar{D}$ tachyon and its fluctuations. In a formalism that treats the transverse scalar x^4 , the $U(1) \times U(1)$ gauge fields A_L and A_R , and the $D\bar{D}$ tachyon \mathcal{T} as the fundamental degrees of freedom one would find four different sectors of mesons: vector mesons (from the fluctuations of $A_+ = A_L + A_R$), axial vector mesons (from the fluctuations of the transverse part of $A_- = A_L - A_R$), pseudoscalar mesons (from the fluctuations of the transverse scalar x^4 and the longitudinal part of A_-) and scalar mesons (from the fluctuations of the $D\bar{D}$ tachyon \mathcal{T}) [12, 8, 9].

In the TDBI language we are instructed to compute instead fluctuations of a real tachyon $T(u, \sigma)$, a transverse scalar $x^4(u, \sigma)$ and a $U(1)$ gauge field $A(u, \sigma)$. The fluctuations of T and x^4 can be viewed as fluctuations of an auxiliary 2-surface (with a cigar topology) in the extended three-dimensional space (T, u, x^4) . The above four mesonic sectors will arise in this language in sectors with different σ -dependence.

The TDBI properties of D8-brane dynamics in the Sakai-Sugimoto model will be discussed more extensively in [21].

6. Conclusions

Hairpin-branes appear frequently in holographic contexts that exhibit flavor chiral symmetry breaking. We argued that the low-energy spectrum of the open string theory on these branes includes, besides the standard gauge field and transverse scalars, a light complex scalar field. Following the setup of Ref. [14] we explored whether the abelian tachyon-DBI action can describe the low-energy dynamics on hairpin-branes capturing efficiently the non-local dynamics of a bifundamental complex scalar field on them. Our main purpose in this paper was to test how well this effective field theory performs in setups where open string theory can be solved with α' -exact methods.

We focused on the $NS5-Dp-\overline{Dp}$ system in a regime that is captured by hairpin Dp -branes in the near-horizon region of the Wick-rotated black NS5-brane. We showed that a radially condensing tachyon-paperclip solution of the abelian tachyon-DBI action gives a satisfactory description of some of the key features of the exact string solution and used the available information from string theory to put constraints on the tachyon potential $V(T)$ (under a certain assumption for the tachyon-paperclip solution). These results can be used to study further, within a well motivated effective field theory, deformations of the standard hairpin-

branes where the non-normalizable branch of the $D\bar{D}$ tachyon is also turned on.

A major motivation behind this work has been to understand better similar situations in holographic backgrounds with RR fields where an explicit open string description is currently out of reach. D8 hairpin-branes in the Sakai-Sugimoto model for QCD is one of these cases. One would like to apply the above framework in these more complicated situations, establish the corresponding rules of holography and obtain new lessons for the flavor dynamics of strongly coupled large- N gauge theories. Phenomenologically interesting quantities like the spectrum of mesons should be computed now under a new prism [21].

Besides the very interesting applications to gauge theory we believe that this exercise will also be useful in uncovering new information about open string dynamics in curved backgrounds. One would like to test the applicability of the general action (2.1) in diverse situations and derive appropriate constraints on the tachyon potential $V(T)$.

Acknowledgements

I am grateful to Vincenzo Calò for the collaboration during the initial stages of this work, and to Emilian Dudas, Elias Kiritsis, David Mateos, Angel Paredes and Ashoke Sen for instructive discussions. This work has been supported by the European Union through an Individual Marie Curie Intra-European Fellowship. Additional support was provided by the ANR grant, ANR-05-BLAN-0079-02, the RTN contracts MRTN-CT-2004-005104 and MRTN-CT-2004-503369, and the CNRS PICS # 3059, 3747, and 4172.

Appendices

A. Worldsheet boundary interactions for hairpin-branes on the cigar

In this appendix we review the worldsheet boundary interactions that characterize hairpin-branes on the $\mathcal{N} = (2, 2)$ supersymmetric $SL(2)_k/U(1)$ coset following the analysis of [18] and derive the first relation in eq. (3.23).

The hairpin-branes of the supersymmetric coset are related by mirror symmetry to the B-type branes of $\mathcal{N} = 2$ Liouville theory that were analyzed in [18]. The worldsheet boundary interactions characterizing these branes are discussed in section 5.1 of that paper. They involve two types of boundary contributions to the worldsheet action. The first one is

$$\oint dx \left[\bar{\lambda} \partial_x \lambda - (\mu_B \lambda + \mu_{\bar{B}} \bar{\lambda}) e^{-\frac{k}{2}(\rho + i\tilde{\theta}) + iH} - (\bar{\mu}_B \lambda + \bar{\mu}_{\bar{B}} \bar{\lambda}) e^{-\frac{k}{2}(\rho - i\tilde{\theta}) - iH} \right] \quad (\text{A.1})$$

where $\lambda, \bar{\lambda}$ are boundary fermions and $\mu_B, \mu_{\bar{B}}, \bar{\mu}_B, \bar{\mu}_{\bar{B}}$ are the corresponding boundary couplings and the second one is

$$- \oint dx \tilde{\mu} (\lambda \bar{\lambda} - \bar{\lambda} \lambda) (\partial H - k \partial \theta) e^{-\rho} \quad (\text{A.2})$$

where $\tilde{\mu}$ is the corresponding boundary coupling.

Using the necessary bootstrap techniques Ref. [18] shows that the boundary couplings can be expressed in terms of the brane labels (J, M) and bulk coupling μ_{bulk} in the following way

$$(\mu_B, \mu_{\bar{B}}, \bar{\mu}_B, \bar{\mu}_{\bar{B}}) = i \sqrt{\frac{k \mu_{bulk}}{2\pi}} \left(e^{i\pi(J-M)}, e^{-i\pi(J-M)}, e^{-i\pi(J+M)}, e^{i\pi(J+M)} \right), \quad (\text{A.3})$$

$$\tilde{\mu} = -i \sin \left(\frac{\pi}{k} (2J - 1) \right) \frac{\mu_{bulk}^{\frac{1}{k}} \Gamma \left(-\frac{1}{k} \right)}{2\pi k}. \quad (\text{A.4})$$

Setting $J = \frac{1}{2} + iP$ and $M = 0$ we obtain

$$(\mu_B, \mu_{\bar{B}}, \bar{\mu}_B, \bar{\mu}_{\bar{B}}) = \sqrt{\frac{k \mu_{bulk}}{2\pi}} \left(-e^{-\pi P}, e^{\pi P}, e^{\pi P}, -e^{-\pi P} \right) \quad (\text{A.5})$$

$$\tilde{\mu} = \frac{\mu_{bulk}^{\frac{1}{k}} \Gamma \left(-\frac{1}{k} \right)}{2\pi k} \sinh \left(\frac{2\pi P}{k} \right). \quad (\text{A.6})$$

For non-zero P we can rotate the boundary fermions

$$\mu \xi = \mu_B \lambda + \mu_{\bar{B}} \bar{\lambda}, \quad \bar{\mu} \bar{\xi} = \bar{\mu}_B \lambda + \bar{\mu}_{\bar{B}} \bar{\lambda} \quad (\text{A.7})$$

with

$$\mu \bar{\mu} = \bar{\mu}_B \mu_{\bar{B}} - \mu_B \bar{\mu}_{\bar{B}} \quad (\text{A.8})$$

to recast the boundary interaction (A.1) into the form

$$\oint dx \left[\bar{\xi} \partial_x \xi - \mu \xi e^{-\frac{k}{2}(\rho + i\tilde{\theta}) + iH} - \bar{\mu} \bar{\xi} e^{-\frac{k}{2}(\rho - i\tilde{\theta}) - iH} \right]. \quad (\text{A.9})$$

These relations define the boundary couplings $\mu, \bar{\mu}$ that appear in the main text (eqs. (3.22), (3.23)). Inserting the values (A.5) into (A.8) we obtain

$$\mu \bar{\mu} = \frac{k \mu_{bulk}}{\pi} \sinh(2\pi P) \quad (\text{A.10})$$

reproducing the first relation in (3.23).

Observe that $P = 0$ is a special case for this manipulation. In that case, the transformation $(\lambda, \bar{\lambda}) \rightarrow (\xi, \bar{\xi})$ (A.7) is singular. Setting directly $P = 0$ in (A.1) one finds the boundary interaction

$$\oint dx \left[\bar{\lambda} \partial_x \lambda - \sqrt{\frac{k \mu_{bulk}}{2\pi}} (-\lambda + \bar{\lambda}) e^{-\frac{k}{2}(\rho + i\tilde{\theta}) + iH} + \sqrt{\frac{k \mu_{bulk}}{2\pi}} (-\lambda + \bar{\lambda}) e^{-\frac{k}{2}(\rho - i\tilde{\theta}) - iH} \right] \quad (\text{A.11})$$

which sets the boundary couplings of the vertex operators $e^{-\frac{k}{2}(\rho \pm i\tilde{\theta}) \pm iH}$ in a different fashion.

B. Tachyon-paperclips and the $D\bar{D}$ tachyon in general holographic backgrounds

In this appendix we consider the general setup of a holographic background with a radial coordinate u and an angular coordinate θ . We restrict attention to a diagonal background metric that depends only on u and analyze the TDBI action for a non-BPS $D(p+1)$ -brane oriented along a set of directions x^μ ($\mu = 0, 1, \dots, p-1$), that exhibit Lorentz invariance, and u, θ . The tachyon potential $V(T)$ is chosen to have the generic form of Fig. 4. We assume that the corresponding equations of motion have a tachyon-paperclip solution $T = T(u, \theta; \phi)$ at large u that depends on a free parameter ϕ , *i.e.* ϕ is a modulus of the solution.

Let us promote ϕ to a slowly-varying field that depends on the coordinates x^μ

$$\phi = \phi(x^\mu) . \quad (\text{B.1})$$

Inserting the asymptotic solution $T(u, \theta; \phi) \equiv T_\phi$ into the TDBI action we expand up to quadratic order in derivatives $\partial_\mu \phi$. Integrating out the u, θ dependence and redefining ϕ to a canonically normalized field ϕ_{can} we obtain the kinetic term

$$\mathcal{S}_{kin} \sim \int dx^0 \dots dx^{p-1} (\partial_\mu \phi_{can})^2 \quad (\text{B.2})$$

and no potential terms. The absence of potential terms is a consequence of the assumption that ϕ is a modulus of the asymptotic paperclip solution.

According to the discussion in the main text the paperclip solution captures implicitly the physics a hairpin Dp -brane oriented along the spacetime directions x^μ, u . In this appendix we are interested in identifying the relation between the hairpin-brane $D\bar{D}$ tachyon \mathcal{T} and the TDBI tachyon T .

For large positive T and real \mathcal{T} we postulate the following relation between \mathcal{T} and ϕ_{can}

$$\mathcal{T} \sim \phi_{can}^\zeta . \quad (\text{B.3})$$

ζ is a constant related to the $U(1)_L - U(1)_R$ charge of ϕ_{can} . \mathcal{T} is implicitly defined here with an appropriate u -dependent normalization factor that cancels its leading u -dependence. In flat space [14] there is no u -dependence and

$$\phi_{can} \sim e^{-\frac{\mathcal{T}}{2\sqrt{2}}} , \quad \zeta = 2 . \quad (\text{B.4})$$

To identify the kinetic term (B.2) we insert the asymptotic solution T_ϕ into the TDBI action and expand to quadratic order. The TDBI action reads

$$\mathcal{S} = - \int dx^0 \dots dx^{p-1} du d\theta V(T) \sqrt{\det A} \quad (\text{B.5})$$

with background metric

$$ds^2 = dT^2 + g_{\mu\nu}(u)dx^\mu dx^\nu + g_{uu}(u)du^2 + g_{\theta\theta}(u)d\theta^2 \quad (\text{B.6})$$

and

$$A_{\mu\nu} = g_{\mu\nu} + (\partial_\phi T)^2 \partial_\mu \phi \partial_\nu \phi , \quad (\text{B.7a})$$

$$A_{\mu u} = \partial_\phi T \partial_u T \partial_\mu \phi , \quad A_{\mu\theta} = \partial_\phi T \partial_\theta T \partial_\mu \phi , \quad (\text{B.7b})$$

$$A_{uu} = g_{uu} + (\partial_u T)^2 , \quad A_{\theta\theta} = g_{\theta\theta} + (\partial_\theta T)^2 , \quad A_{u\theta} = \partial_u T \partial_\theta T . \quad (\text{B.7c})$$

By assumption

$$g_{\mu\nu}(u) = g(u)\eta_{\mu\nu} . \quad (\text{B.8})$$

Let us denote by \mathcal{A} the $p \times p$ matrix with elements $A_{\mu\nu}$, by \mathcal{B} the $p \times 2$ matrix $(A_{\mu u}, A_{\mu\theta})$, by \mathcal{B}^T its transverse and by \mathcal{C} the 2×2 matrix

$$\begin{pmatrix} A_{uu} & A_{u\theta} \\ A_{u\theta} & A_{\theta\theta} \end{pmatrix} . \quad (\text{B.9})$$

A useful identity for the computation of $\det A$ is

$$\det \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B}^T & \mathcal{C} \end{pmatrix} = \det \mathcal{A} \cdot \det (\mathcal{C} - \mathcal{B}^T \mathcal{A}^{-1} \mathcal{B}) . \quad (\text{B.10})$$

After some algebra we find

$$\det A = g \det \mathcal{C} - (\det \mathcal{C} - g_{uu}g_{\theta\theta}) (\partial_\phi T)^2 (\partial_\mu \phi)^2 . \quad (\text{B.11})$$

Expanding the square root

$$\sqrt{\det A} \simeq \sqrt{g \det \mathcal{C}} - \frac{1}{2} \frac{\det \mathcal{C} - g_{uu}g_{\theta\theta}}{\sqrt{g \det \mathcal{C}}} (\partial_\phi T)^2 (\partial_\mu \phi)^2 + \dots . \quad (\text{B.12})$$

The diverging factor

$$\mathcal{K}(\phi) = \int du d\theta V(T_\phi) \frac{\det \mathcal{C} - g_{uu}g_{\theta\theta}}{\sqrt{g \det \mathcal{C}}} (\partial_\phi T_\phi)^2 \quad (\text{B.13})$$

is a function of ϕ whose computation requires explicit information about the tachyon potential and the tachyon-paperclip solution. In terms of $\mathcal{K}(\phi)$ the kinetic term (B.2) becomes

$$\mathcal{S}_{kin} \sim -\frac{1}{2} \int dx^0 \dots dx^{p-1} \mathcal{K}(\phi) (\partial_\mu \phi)^2 \quad (\text{B.14})$$

from which we can read off ϕ_{can} .

For the tachyon-paperclip solution of the NS5-D p - $\overline{\text{D}}p$ system (4.18) we have (renaming u as ρ)

$$g = 1, \quad T_\phi = \phi + b\rho + c \log \cos \theta, \quad \det \mathcal{C} = \frac{kc^2}{\cos^2 \theta}, \quad V(T) = e^{-\beta T}. \quad (\text{B.15})$$

Hence, up to a ϕ -independent diverging factor that comes from the ρ, θ integration

$$\mathcal{K}(\phi) \sim e^{-\beta\phi} \quad (\text{B.16})$$

and

$$\phi_{can} \sim e^{-\frac{\beta}{2}\phi}, \quad (\text{B.17})$$

which together with (B.3) implies the transformation (4.26) of section 4.

C. Further numerical results on the relation $\mu_T(\rho_0)$: the case of generic k

Note added: This appendix has been added to the second arXiv version of the paper in September of 2010 and does not appear in the published Nuclear Physics B version which appeared earlier.

Following the logic outlined in the beginning of section 4.6, we consider here a differential equation that provides an estimate to the behavior of the tachyon maximum $T(\rho, 0)$ for generic k . We assume that $T \gg 1$ and approximate the tachyon potential by the large- T asymptotics (4.17). It turns out that this approximation does not affect the final result significantly—a direct comparison to the results obtained in section 4.6 for $k = 2$ with the full potential $\cosh^{-1}(T/\sqrt{2})$ reveals small numerical differences.

In what follows we insert the ansatz

$$T(\rho, \theta) = f(\rho) + c \log \cos \theta \quad (\text{C.1})$$

into the equation of motion (4.4), expand to leading order around $\theta = 0$ and obtain the differential equation

$$\left(f' \cosh \rho - \left(\frac{c \cosh^2 \rho}{\sinh \rho} - k\beta \sinh \rho \right) \right) (k + f'^2) + k \sinh \rho f'' = 0. \quad (\text{C.2})$$

We are leaving β as a free constant and express c in terms of β using the equations (4.20), (4.21)

$$c = \frac{1}{2} \left(\frac{1}{\beta} + \beta k \right). \quad (\text{C.3})$$

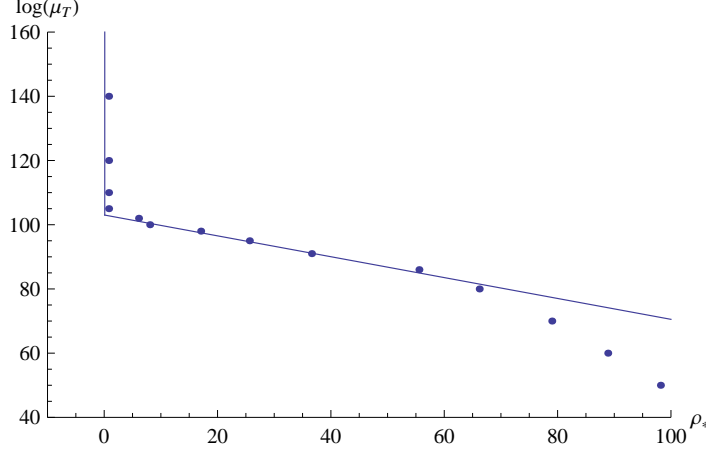


Figure 8: Fourteen numerically determined values of $\log(\mu_T)$ versus ρ_* are plotted in this figure by evaluating the solution of eq. (C.4) for $k = 10$ and $M_T = e^{150}$. The solid curve is a fit based on the functional form (4.47) with $y = 6.5$ and $z = 0.05$.

In the special case of $\beta = \frac{1}{\sqrt{k}}$ (see eq. (4.32)) the equation (C.2) becomes

$$\left(f' \cosh \rho - \frac{\sqrt{k}}{\sinh \rho} \right) (k + f'^2) + k \sinh \rho f'' = 0 \quad (\text{C.4})$$

which reduces correctly to eq. (4.45) for $k = 2$ and $f \gg 1$.

We can solve numerically the differential equation (C.2) for different values of k and β . For each of these values we obtain a curve that expresses, at fixed and large M_T , the relation between the boundary value μ_T and the singularity point ρ_* (which is treated here as an approximation to the turning point of the brane). Analytically this curve is expected to be of the form (4.47), (4.48). We find that the numerically determined curve does not obey this functional relation for generic values of β .¹⁰ A qualitative and quantitative match to the expected analytical result is observed only when β takes the special value $\frac{1}{\sqrt{k}}$ (4.32). We view this result as strong numerical evidence in favor of the expectation expressed below eq. (4.32) that the expressions in (4.32) provide the correct tachyon asymptotics for all values of k .

Fig. 8 depicts the numerically determined curve $\log(\mu_T)$ as a function of ρ_* for a randomly chosen value of k ($k = 10$), the special value of β ($\beta = \frac{1}{\sqrt{10}}$), and $M_T = e^{150}$. The solid curve is a fit based on the analytically expected form (4.47) with fit values $y = 6.5$ and $z = 0.05$. The analytic values of y and z compare well with the fit values—for $k = 10$ these are (see eq. (4.48)) $y = k = 10$ and $z = \frac{1}{18} \simeq 0.0556$. As in section 4.6 we observe an expected

¹⁰For generic values of β there is a shift of the point where μ_T diverges away from the tip of the cigar.

mismatch between the fit and the numerically determined points in the vicinity of the point where the boundary conditions are placed (here $\rho = 100$).

Similar features are observed for generic values of k . The agreement between the fit value and the analytical value of the slope of the linear section of the curve, *i.e.* the parameter yz , appears to improve as we increase k . For $k = 2$ (see e.g. section 4.6) we obtain $(yz)_{num} \simeq 0.4$ versus $(yz)_{anal} = 1$. For $k = 10$ we obtain $(yz)_{num} \simeq 0.325$ versus $(yz)_{anal} = \frac{10}{18} \simeq 0.556$.

References

- [1] M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, “Towards a holographic dual of large- $N(c)$ QCD,” JHEP **0405**, 041 (2004) [arXiv:hep-th/0311270].
- [2] A. Karch and E. Katz, “Adding flavor to AdS/CFT,” JHEP **0206**, 043 (2002) [arXiv:hep-th/0205236].
- [3] T. Sakai and S. Sugimoto, “Low energy hadron physics in holographic QCD,” Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141].
- [4] T. Sakai and S. Sugimoto, “More on a holographic dual of QCD,” Prog. Theor. Phys. **114**, 1083 (2005) [arXiv:hep-th/0507073].
- [5] O. Aharony, “A brief review of ‘little string theories’,” Class. Quant. Grav. **17**, 929 (2000) [arXiv:hep-th/9911147].
- [6] E. Antonyan, J. A. Harvey, S. Jensen and D. Kutasov, “NJL and QCD from string theory,” arXiv:hep-th/0604017.
- [7] A. Fotopoulos, V. Niarchos and N. Prezas, “D-branes and SQCD in non-critical superstring theory,” JHEP **0510**, 081 (2005) [arXiv:hep-th/0504010].
- [8] O. Bergman, S. Seki and J. Sonnenschein, “Quark mass and condensate in HQCD,” JHEP **0712**, 037 (2007) [arXiv:0708.2839 [hep-th]].
- [9] A. Dhar and P. Nag, “Tachyon condensation and quark mass in modified Sakai-Sugimoto model,” Phys. Rev. D **78**, 066021 (2008) [arXiv:0804.4807 [hep-th]].
- [10] A. Sen, “Dirac-Born-Infeld action on the tachyon kink and vortex,” Phys. Rev. D **68** (2003) 066008 [arXiv:hep-th/0303057].
- [11] M. R. Garousi, “D-brane anti-D-brane effective action and brane interaction in open string channel,” JHEP **0501** (2005) 029 [arXiv:hep-th/0411222].
- [12] R. Casero, E. Kiritsis and A. Paredes, “Chiral symmetry breaking as open string tachyon condensation,” Nucl. Phys. B **787**, 98 (2007) [arXiv:hep-th/0702155].
- [13] I. Iatrakis, E. Kiritsis and A. Paredes, “An AdS/QCD model from Sen’s tachyon action,”

- arXiv:1003.2377 [hep-ph].
- [14] D. Erkal, D. Kutasov and O. Lunin, “Brane-Antibrane Dynamics From the Tachyon DBI Action,” arXiv:0901.4368 [hep-th].
 - [15] D. Kutasov and V. Niarchos, “Tachyon effective actions in open string theory,” Nucl. Phys. B **666**, 56 (2003) [arXiv:hep-th/0304045].
 - [16] V. Niarchos, “Notes on tachyon effective actions and Veneziano amplitudes,” Phys. Rev. D **69**, 106009 (2004) [arXiv:hep-th/0401066].
 - [17] S. L. Lukyanov, E. S. Vitchev and A. B. Zamolodchikov, “Integrable model of boundary interaction: The paperclip,” Nucl. Phys. B **683**, 423 (2004) [arXiv:hep-th/0312168].
 - [18] K. Hosomichi, “N=2 Liouville Theory with Boundary,” JHEP **0612**, 061 (2006) [arXiv:hep-th/0408172].
 - [19] D. Kutasov, “Accelerating branes and the string / black hole transition,” arXiv:hep-th/0509170.
 - [20] S. L. Lukyanov and A. B. Zamolodchikov, “Dual form of the paperclip model,” Nucl. Phys. B **744**, 295 (2006) [arXiv:hep-th/0510145].
 - [21] V. Niarchos, “Notes on a Novel Description of Flavor Dynamics in the Sakai-Sugimoto Model,” work in progress.
 - [22] A. Sen, “Tachyon dynamics in open string theory,” Int. J. Mod. Phys. A **20**, 5513 (2005) [arXiv:hep-th/0410103].
 - [23] Y. Nambu and G. Jona-Lasinio, “Dynamical model of elementary particles based on an analogy with superconductivity. I,” Phys. Rev. **122**, 345 (1961).
 - [24] C. G. . Callan, J. A. Harvey and A. Strominger, “Supersymmetric string solitons,” arXiv:hep-th/9112030.
 - [25] S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici and G. Sarkissian, “D-branes in the background of NS fivebranes,” JHEP **0008**, 046 (2000) [arXiv:hep-th/0005052].
 - [26] D. Israel, A. Pakman and J. Troost, “D-branes in little string theory,” Nucl. Phys. B **722**, 3 (2005) [arXiv:hep-th/0502073].
 - [27] E. Witten, “On string theory and black holes,” Phys. Rev. D **44**, 314 (1991).
 - [28] A. Fotopoulos, “Semiclassical description of D-branes in $SL(2)/U(1)$ gauged WZW model,” Class. Quant. Grav. **20**, S465 (2003) [arXiv:hep-th/0304015].
 - [29] I. Antoniadis, E. Dudas and A. Sagnotti, “Supersymmetry breaking, open strings and M-theory,” Nucl. Phys. B **544**, 469 (1999) [arXiv:hep-th/9807011].
 - [30] S. Ribault and V. Schomerus, “Branes in the 2-D black hole,” JHEP **0402**, 019 (2004) [arXiv:hep-th/0310024].

- [31] T. Eguchi and Y. Sugawara, “Modular bootstrap for boundary $N = 2$ Liouville theory,” JHEP **0401**, 025 (2004) [arXiv:hep-th/0311141].
- [32] C. Ahn, M. Stanishkov and M. Yamamoto, “One-point functions of $N = 2$ super-Liouville theory with boundary,” Nucl. Phys. B **683**, 177 (2004) [arXiv:hep-th/0311169].
- [33] D. Israel, A. Pakman and J. Troost, “D-branes in $N = 2$ Liouville theory and its mirror,” Nucl. Phys. B **710**, 529 (2005) [arXiv:hep-th/0405259].
- [34] C. Ahn, M. Stanishkov and M. Yamamoto, “ZZ-branes of $N = 2$ super-Liouville theory,” JHEP **0407**, 057 (2004) [arXiv:hep-th/0405274].
- [35] A. Fotopoulos, V. Niarchos and N. Prezas, “D-branes and extended characters in $SL(2, \mathbb{R})/U(1)$,” Nucl. Phys. B **710**, 309 (2005) [arXiv:hep-th/0406017].
- [36] Y. Kazama and H. Suzuki, “New $N=2$ Superconformal Field Theories and Superstring Compactification,” Nucl. Phys. B **321**, 232 (1989).
- [37] A. Giveon and D. Kutasov, “Little string theory in a double scaling limit,” JHEP **9910**, 034 (1999) [arXiv:hep-th/9909110].
- [38] K. Hori and A. Kapustin, “Duality of the fermionic 2d black hole and $N = 2$ Liouville theory as mirror symmetry,” JHEP **0108**, 045 (2001) [arXiv:hep-th/0104202].
- [39] I. Bars and K. Sfetsos, “Conformally exact metric and dilaton in string theory on curved space-time,” Phys. Rev. D **46**, 4510 (1992) [arXiv:hep-th/9206006].
- [40] A. A. Tseytlin, “Conformal sigma models corresponding to gauged Wess-Zumino-Witten theories,” Nucl. Phys. B **411**, 509 (1994) [arXiv:hep-th/9302083].
- [41] C. Ahn, C. Kim, C. Rim and M. Stanishkov, “Duality in $N = 2$ super-Liouville theory,” Phys. Rev. D **69**, 106011 (2004) [arXiv:hep-th/0210208].
- [42] Y. Nakayama, “Liouville field theory: A decade after the revolution,” Int. J. Mod. Phys. A **19**, 2771 (2004) [arXiv:hep-th/0402009].
- [43] E. Witten, “On background independent open string field theory,” Phys. Rev. D **46**, 5467 (1992) [arXiv:hep-th/9208027].
- [44] D. Kutasov, M. Marino and G. W. Moore, “Some exact results on tachyon condensation in string field theory,” JHEP **0010**, 045 (2000) [arXiv:hep-th/0009148].
- [45] D. Kutasov and N. Seiberg, “More comments on string theory on $AdS(3)$,” JHEP **9904**, 008 (1999) [arXiv:hep-th/9903219].
- [46] D. Kutasov, “A geometric interpretation of the open string tachyon,” arXiv:hep-th/0408073.